ANALYSIS QUALIFYING EXAM

FEBRUARY 13, 2002

Problem	Score
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Total	

Last Name:

First Name:

Social Security Number:

If you are taking the Real Analysis (525a) exam only: Do problems #1–4.

If you are taking the Complex Analysis (520) exam only: Do problems #5–8.

If you are taking both parts: In order to pass, you must do well on both the Real and Complex Analysis parts—high performance on one portion does not compensate for low performance on the other.

Do as many problems as you can.

REAL ANALYSIS PROBLEMS

1. A mapping $f : \mathbb{R}^N \to \mathbb{R}^N$ is said to be *Lipschitzian* if there exists a constant M such that

$$\|f(x) - f(y)\| \leq M\|x - y\|$$

for all $x, y \in \mathbb{R}^N$. Prove: If $f : \mathbb{R}^N \to \mathbb{R}^N$ is Lipschitzian and $\Omega \subset \mathbb{R}^N$ is Lebesgue measurable, then $f(\Omega)$ is also Lebesgue measurable.

2. Let $(X, \mathcal{A}, \mu)$ be a measure space and $\{f_n\}$ a sequence of nonnegative $\mathcal{A}$-measurable functions which converges μ-a.e. to 0. Suppose there exists a finite constant M such that

$$\int \max\{f_1, \ldots, f_n\}\, d\mu \leq M$$

for all n. Prove: $\int f_n\, d\mu \to 0$ as $n \to \infty$.

3. Let $f \in L^1(X, \mathcal{A}, \mu)$. Prove:

$$\|f\|_1 = \int_0^\infty \mu\big(\{x : |f(x)| \geq \lambda\}\big)\, d\lambda.$$

4. Let f be continuous on $[-1, 1]$. Show that

$$\lim_{n\to\infty} n \int_{-1/n}^{1/n} f(x)\big(1 - n|x|\big)\, dx$$

exists, and evaluate it.