MATH 505a **QUALIFYING EXAM** **February 03, 2005**

Last Name: _____ **First Name:** _____

ID#: _____ **Signature:** _____

You should try at least three problems; you may try all four.

1. Let $X_1, X_2, \ldots, X_n$ be a sample of independent, identically distributed random variables with density f and distribution function F. Let $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ be the ordered sequence of those variables. For $i < j$ find the joint density of $(X_{(i)}, X_{(j)})$.

2. Let N be a Poisson random variable with parameter λ. Let $Y = \sum_{i=1}^{N} X_i$, where X_i are independent, identically distributed, non-negative integer valued random variables with finite mean. Show that for any function g (such that the expectations exist) we have

$$E[Y g(Y)] = \lambda E[X_0 g(Y + X_0)].$$

3. A stick is broken in two pieces, uniformly at random. Let X denote the ratio of the lengths of the shorter to the longer piece. Find the mean and the variance of X.

4. The number of the electrons that hit the plate is Poisson with parameter $\lambda_1 = 2$. Every impact produces independently a number of secondary electrons that is Poisson with parameter $\lambda_2 = 1$. a) Find the moment generating function of the total number of secondary electrons; b) Find the variance of that number.