# MATH 505a QUALIFYING EXAM
## January 27, 2004

You should try at least three problems; you may try all four.

**Problem 1.** Consider four points, 1,2,3 and 4. For each two points, there is a link between them with probability $p$, and no link with probability $1-p$, independently of other points. What is the probability that there is a path of connected links between points 1 and 2?

**Problem 2.** Let $X$ be a non-negative random variable with finite expectation. Show that

$$\sum_{i=1}^{\infty} P(X \geq i) \leq E[X] < 1 + \sum_{i=1}^{\infty} P(X \geq i).$$

**Problem 3.** Let $X$ and $Y$ be random variables with means equal to zero, variances equal to one, and correlation coefficient $\rho$. Show that

$$\left(E[\sqrt{\max(X^2, Y^2)}]\right)^2 \leq 1 + \sqrt{1-\rho^2} \ .$$

Hint: First show that $\max(x, y) = (x + y + |x - y|)/2$.

**Problem 4.** Let $\phi(t)$ be a characteristic function of a random variable $X$.
a) Assume $\phi(t) = e^{iat}, t \in (-\infty, \infty)$. Show that $\mathbf{P}$-a.s. $X = a$.
b) Assume that $\phi(2\pi) = 1$. Show that

$$\sum_{k=-\infty}^{\infty} P(X = k) = 1.$$