## Math 505a Exam Portion. Spring 2001

**Problem 1.**

(i) Each person in a group of $n$ individuals is assigned a different number between 1 and $n$. After that, each person randomly picks a number between 1 and $n$ from a box. What is the expected value of the number of persons who pick the same number as the one originally assigned to them?

(ii) Consider continuing this game as follows: Suppose that when a person picks the same number as the one originally assigned to them, they stop playing and take the number with them. Those who have not picked their assigned number, try again in the next round, with the remaining numbers in the box. Show that the expected number of rounds necessary for everyone to pick their assigned number is equal to $n$.

**Problem 2.** Let $X_1, X_2$ be normal random variables with mean zero. Suppose $Var(X_1) = 1$ and $Var(X_2) = 2$. Suppose that $X_2 - X_1$ is independent of $X_1$. Denote, for a given $y > 0$, $\sigma > 0$,

$$Y = ye^{\sigma X_2 - \sigma^2}.$$

a) Compute $E[Y]$.

b) Given $K > 0$, show that

$$E[Y \mathbf{1}_{\{Y>K\}}] = y\Phi\left(\frac{\log(y/K) + \sigma^2}{\sigma\sqrt{2}}\right),$$

where $\Phi$ is the cumulative standard normal distribution function $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-u^2/2}du$. Here, $\mathbf{1}_A$ is one if $A$ occurs, and is zero if it does not occur.

c) Find a similar expression for the conditional expectation $E[Y\mathbf{1}_{\{Y>K\}}|X_1]$.