

MATH 505a QUALIFYING EXAM  
September 20, 2004

You should try at least three problems; you may try all four.

1. There are three coins that show heads with probability  $2/3$ , and tails otherwise. The first coin counts 8 points for a head and 3 for a tail, the second counts 5 points for both head and tail, and the third counts 4 points for a head and 10 for a tail. You and your opponent choose a coin, and you cannot choose the same coin. Each of you tosses your coin and the person with the larger score wins. Would you prefer to be the first to pick a coin, or the second?
2. A coin with probability  $p$  for heads is tossed  $n$  times. Find the expected number of "runs" and the variance of the number of "runs", where "run" is a sequence of identical outcomes. For example, the sequence  $TTTHHTHH$  has four runs,  $TTT$ ,  $H$ ,  $T$  and  $HH$ .
3. Let  $X_n$  be a sequence of independent identical distributed random variables with distribution function  $F$  such that  $F(x) < 1$  for all  $x$ . Let  $Y(y) = \min\{k : X_k > y\}$ . Find the probability

$$P(Y(y) \leq E[Y(y)]).$$

Can you also show that the limit of this probability when  $y \rightarrow \infty$  is  $1 - 1/e$ ?

4. Let  $X_1, X_2, \dots, X_n$  be independent identically distributed normal random variables with mean  $\mu$  and variance  $\sigma^2$ . Consider

$$S_n = X_1 + \dots + X_n.$$

- Find the moment generating function of  $S_n$ .
- Find  $E(e^{S_n})$ .
- What is the moment generating function of  $(S_n - n\mu)/\sqrt{n}\sigma$ ?