

MATH 505a QUALIFYING EXAM
September 20, 2004

You should try at least three problems; you may try all four.

1. There are three coins that show heads with probability $2/3$, and tails otherwise. The first coin counts 8 points for a head and 3 for a tail, the second counts 5 points for both head and tail, and the third counts 4 points for a head and 10 for a tail. You and your opponent choose a coin, and you cannot choose the same coin. Each of you tosses your coin and the person with the larger score wins. Would you prefer to be the first to pick a coin, or the second?

2. A coin with probability p for heads is tossed n times. Find the expected number of "runs" and the variance of the number of "runs", where "run" is a sequence of identical outcomes. For example, the sequence $TTTHTHH$ has four runs, TTT , H , T and HH .

3. Let X_n be a sequence of independent identically distributed random variables with distribution function F such that $F(x) < 1$ for all x . Let $Y(y) = \min\{k : X_k > y\}$. Find the probability

$$P(Y(y) \leq E[Y(y)]).$$

Can you also show that the limit of this probability when $y \rightarrow \infty$ is $1 - 1/e$?

4. Let X_1, X_2, \dots, X_n be independent identically distributed normal random variables with mean μ and variance σ^2 . Consider

$$S_n = X_1 + \dots + X_n.$$

- a) Find the moment generating function of S_n .
- b) Find $E(e^{S_n})$.
- c) What is the moment generating function of $(S_n - n\mu)/\sqrt{n}\sigma$?