Math 505 Exam

Problem 1: Let $X_0, X_1, \ldots, X_n$ be iid variables with continuous distribution. Let

$$N = \inf\{n \geq 1 : X_n > X_0\}$$

a) Find the conditional distribution of N given X_0,

$$P(N = n | X_0 = x).$$

b) Find the (conditional) mean $E(N|X_0)$ of this distribution.

c) Compute $P(N = n)$ using

$$P(N = n) = E\{P(N = n|X_0)\}$$

d) Compute EN using

$$EN = E\{E(N|X_0)\}$$

e) Interpret the results of c) and d)

Problem 2. Let Y and U be two independent random variables with $Y \sim \mathcal{N}(0,1)$ and $P(U = 1) = P(U = -1) = 1/2$. Let $Z = UY$. Show that

a) $Z \sim \mathcal{N}(0,1)$;

b) Y and Z are uncorrelated.

c) Y and Z are **not** independent.

Problem 3. Let S_n be simple symmetric random walk. a) Show that

$$P(S_1 S_2 \ldots S_{2n} \neq 0) = P(S_{2n} = 0) \quad n \geq 1.$$

You may use any method of your choice (e.g. generating functions, one to one path correspondences, etc.)

b) Compute the probability in a).