

# COMPLEX ANALYSIS GRADUATE EXAM

Fall 2023

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Evaluate

$$\int_0^{\infty} \frac{dx}{x^4 + 1}.$$

2. Assume that  $a \geq 1$  and  $b > 2$ . Prove that the equation

$$az^3 - z + b = e^{-z}(z + 2)$$

has two solutions in  $\{z : \operatorname{Re} z > 0\}$ .

3. Prove that the regions  $\{z \in \mathbb{C} : 0 < |z| < 1\}$  and  $\{z \in \mathbb{C} : 1 < |z| < 2\}$  are not conformally equivalent.

4. Assume that an entire function  $f$  maps a circle in a complex plane to  $\mathbb{R}$ . Prove that  $f$  is a constant.