Fall 2008 Math 541a Exam

1. Let $p \in (0,1)$ and $q = 1 - p$.

    (a) Show that

    $$P(X = -1) = p \quad \text{and} \quad P(X = k) = q^2 p^k, \quad k = 0, 1, \ldots$$

    defines a probability distribution for the random variable X.

    (b) Given the single observation X, the statistic X is sufficient; is X also complete?

    (c) Determine all unbiased estimators of p, given one observation of X from the family above. Hint: Consider $T(X) = \mathbf{1}(X = -1)$.

    (d) Find the UMVU of p, or prove that it does not exist.

2. Consider the Pareto distribution $P(a, c)$, with positive parameters a and c, whose density function is given by

    $$p(x; a, c) = \frac{ac^a}{x^{a+1}} \quad \text{for } x \geq c.$$

    (a) Verify $p(x; a, c)$ is a density function, and find the associated distribution function.

    (b) When X has density $p(x; a, c)$, determine the distribution of $Y = \log X$.

    (c) Let $X_1, \ldots, X_n$ be a random sample from the Pareto $P(a, c)$ distribution. Find the maximum likelihood estimators $\hat{a}$ and $\hat{c}$ of a and c, respectively.

    (d) Determine the distribution of $\hat{c}$ or of $2na/\hat{a}$.