MATH 505a **QUALIFYING EXAM** **February 5, 2009**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1.) Let $X_1, X_2, \ldots,$ be a sequence of i.i.d. random variables with a continuous distribution function. Let

$$N = \min\{n \geq 2 : \; X_n > X_{n-1}\}.$$

a) Find $\mathbb{P}(N = n)$ for $n = 2, 3, \ldots$.
b) Find $\mathbb{P}(N \geq n)$ for $n = 1, 2, 3, \ldots$.
c) Find and simplify EN.

2.) A deck (deck #1) of cards has a red cards and b black cards, another deck (deck #2) has c red cards and d black cards. Assume $a, b, c, d \geq 1$. Both decks are well-shuffled. Suppose that you pick f cards ($0 \leq f \leq a + b$) randomly from deck #1 and mix them into deck #2. One card is now selected and removed from the mixed up deck #2.
a) What is the chance that the first card selected is red?
b) Given that the first selected card is red, what is the chance that it originally came from deck #1 ?
c) Are the two events, that the first selected card is red, and that the first selected card originally came from deck #1, independent? [Possible answers include YES, NO, or some relation among the parameters a, b, c, d, f. If you give a relation, please simplify it.]

3.) Suppose that the random variables Y, X, X_1, X_2, $\ldots$, are independent, identically distributed, and that $P\{Y = n\} = 2^{-n}$, for $n = 1, 2, \ldots$; and $P\{X \geq t\} = e^{-\pi t}$, for $t > 0$ and $k = 1, 2, \ldots$. Let $S_n = X_1 + X_2 + \cdots + X_n$. Let $Z = S_Y$.

a) Calculate and simplify $\mathbb{E}\, Z$.
b) Simplify the probability generating function $\mathbb{E}\, s^Y$.
c) Simplify the moment generating function $\mathbb{E}\, \exp(\beta X)$.
d) Simplify $\mathbb{E}\, \exp(\beta Z)$.

e) Calculate and simplify $\mathbb{E}\, Z^3$.

[Hint, if you answered a) by some easy method, use this to check your results for d) and e).]