MATH 505a **QUALIFYING EXAM** **January 31, 2007**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1.) n balls are placed into d boxes at random, with all n^d possibilities equally likely. Assume $d > 8$. Let X be the number of empty boxes. Let D be the event that no box receives more than 1 ball. Let A be the event that boxes 1 and 2 are both empty, B be the event that boxes 3,4,5 are all empty, and C be the event that boxes 6,7,8 are all empty.

a) Calculate and simplify: $\mathbb{E}\,X =$ ______________________

b) Calculate and simplify: $\mathrm{Var\,X} =$ ______________________

c) $\mathbb{P}(A \cup B \cup C) =$ ______________________

d) If both $n, d \to \infty$ together, what relation must they satisfy in order to have $\mathbb{P}(D) \to .1$?

2.) Suppose Z is Poisson with $\mathbb{E}\,Z = \lambda < 1$. Let $X = 2^Z, Y = Z!$. Compute and simplify each of the following:

a) $\mathbb{E}\,Z^2 =$ ______________________
b) $\mathbb{E}\,Z^3 =$ ______________________
c) $\mathbb{E}\,Y =$ ______________________
d) $\mathbb{E}\,X =$ ______________________
e) $\mathrm{Var\,X} =$ ______________________

3) Suppose that X is a sum of indicator random variables, with $\mu = \mathbb{E}\,X = 10, \sigma^2 = \mathrm{Var\,X} = 7$. Let A be the event $\{X > 0\}$.

a) State Chebyshev's inequality, involving the variance and the distance to the mean.
b) Apply Chebyshev's inequality to get a lower bound on $\mathbb{P}(A)$.
c) State the Cauchy-Schwarz for $(\mathbb{E}\,(XY))^2$.
d) Apply Cauchy-Schwarz, with $Y = 1(X > 0)$, the indicator that X is strictly positive, to get a lower bound on $\mathbb{P}(A)$.