1. Suppose that out of n i.i.d. Bernoulli trials, each with probability p of success, there are zero successes.

   (a) Given $\alpha \in (0, 1)$, derive an exact upper $(1 - \alpha)$-confidence bound for p by either pivoting the c.d.f. of the Binomial distribution or inverting the appropriate hypothesis test.

   (b) There is a famous rule of thumb called the "Rule of Threes" which says that, when n is large, $3/n$ is an approximate upper 95%-confidence bound for p in the above situation. Justify the Rule of Threes by applying a large-n first order Taylor approximation to your answer from Part 1a, and use the fact that $|\log(.05)| \approx 3$.

2. Let $w_1, \ldots, w_n$ be i.i.d. from the mixture distribution

$$f(w; \psi) = \sum_{i=1}^{g} \pi_i f_i(w),$$

   where $\psi = (\pi_1, \ldots, \pi_g)$ is a vector of unknown probabilities summing to one, and $f_1, \ldots, f_g$ are *known* density functions.

   (a) Write an equation one would solve to find the maximum likelihood estimate of ψ.

   (b) To implement the EM algorithm, write down the full likelihood when in addition to the sample $w_1, \ldots, w_n$, the 'missing data'

$$Z_{ij} = \mathbf{1}(\text{the } j\text{th observation } w_j \text{ comes from } i\text{th group } f_i),$$

   is also observed.

   (c) Write down the estimate of ψ using the full data likelihood in part (2b).

   (d) Write down the E and M steps of the EM algorithm.