COMPLEX ANALYSIS GRADUATE EXAM

Fall 2016

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Let $A = \{z \in \mathbb{C} : r < |z| < R\}$ for some $0 < r < R < \infty$. Prove that $f(z) = 1/z$ cannot be uniformly approximated in A by complex polynomials.

2. Let $D = \mathbb{C}\backslash[-1, 1]$. Prove that $f(z) = z^2 - 1$, for $z \in D$, has an analytic square root but does not have an analytic logarithm.

3. Evaluate

$$\int_0^\infty \frac{\log x}{1 + x^2}\, dx.$$

4. Show that the range of a nonconstant entire function is dense in $\mathbb{C}$.