

MATH 505a GRADUATE EXAM

Fall 2016

Answer as many questions as you can. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. If you cannot do part (a) of a problem, you can still get credit for (b), (c) etc. by assuming the answer to (a). Start each problem on a fresh sheet of paper, and write on only one side of the paper.

(1) You have a choice to roll a fair die either 100 times or 1000 times. For each of the following outcomes, state whether it is more likely with 100 rolls, or with 1000 rolls. Justify your answer, but you do not need to give a full formal proof.

- (a) The number 1 shows on the die between 15% and 20% of the time.
- (b) The number showing is at most 3, at least half the time.
- (c) The number showing is 2 or 5, at least half the time.

(2) Let X and Y be independent exponential random variables with parameters λ and μ (that is, $E(X) = 1/\lambda$ and $E(Y) = 1/\mu$), and let $Z = \min(X, Y)$.

(a) Show that Z is independent of the event $X < Y$. In other words, show the event $Z \leq t$ is independent of $X < Y$ for all t .

(b) Find the distribution of $\max(X - Y, 0)$.

(3) Let X_1, X_2, \dots be iid with characteristic function φ . Let N be independent of the X_i 's with $P(N = n) = 2^{-n}$ for all $n \geq 1$. Let $Y = \sum_{i=1}^N X_i$. Find the characteristic function of Y .

(4) $n \geq 4$ men, among whom are Alfred, Bill, Charles and David, stand in a row. Assume that all possible orderings of the n men are equally likely.

(a) Find the probability that Charles stands somewhere between Alfred and Bill. (Note this does not mean they are necessarily adjacent—there might be other people between Alfred and Bill.)

(b) Find the probability that David stands somewhere between Alfred and Bill given that Charles stands somewhere between Alfred and Bill.

(c) Find the expected value and variance of the number of men out of n who stand between Alfred and Bill. (Note Alfred and Bill themselves are not counted in this number.)