MATH 505a **QUALIFYING EXAM** **September 23, 2008**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1.) The number of cars arriving at a McDonald's drive-up window in a given day is a Poisson random variable, N, with parameter λ. The numbers of passengers in these cars are independent random variables, X_i, each equally likely to be one, two, three, or four.

a) Simplify the probability generating function of N, say $G_N(z) = \mathbb{E}\, z^N =$ _____.

b) Simplify the moment generating function of $X = X_i$, say $M_X(t) = \mathbb{E}\, e^{tX} =$ _____.

c) Find the moment generating function of the total number of passengers passing by the drive-up window in a given day. (Hint: $S = \sum_{i=1}^{N} X_i$.)

2.) Consider a lottery with n^2 tickets, of which exactly n win prizes. A person buys $2n$ tickets. Find the following limits; part credit for guessing plausibly, and part credit for a proof.[Hint: the lottery involves drawing without replacement, but a good guess arises by thinking of drawing tickets with replacement.]

a) $\lim_{n\to\infty} \mathbb{P}($ at least one winning ticket$) =$ _____

b) $\lim_{n\to\infty} \mathbb{P}($ exactly 3 winning tickets$) =$ _____

3a) Suppose that S and S' are iid standard exponential, and $r > 0$.
Show that $P(rS < S') = 1/(1+r)$. [Hint: you might answer either by a detailed calculation, or by an informal Poisson process argument.]

3b) Find the covariance of (D_1, D_2, D_3, D_4) where $D_1 = (X_2 - X_1)/2, D_2 = (Y_2 - Y_1)/2, D_3 = X_3 - (X_1 + X_2)/2, D_4 = Y_3 - (Y_1 + Y_2)/2$ and the six coordinates (X_i, Y_i) for $i = 1$ to 3 are iid standard normal.

3c). Suppose that A, B, C are points in the plane, whose six coordinates (X_i, Y_i) for $i = 1$ to 3 are iid standard normal. There is a unique circle having the segment from A to B as a diameter. Show that the probability that C lies inside this circle is 1/4. [Hint: 3a) and 3b) are both useful here. Think about the square of the distance from C to the midpoint of A,B, and about the square of the radius of the circle.]