1. For given density functions p and q we consider the Neyman-Pearson tests of level $\alpha \in (0,1)$ for the hypotheses $H_0 : r = p$ versus $H_1 : r = q$, when observing n independent observations from density r.

   (a) Derive the Neyman-Pearson test for a given level $\alpha \in (0,1)$ when p and q are the density functions of the $\mathcal{N}(\mu, \sigma^2)$ distribution with means $\mu = \mu_0$ and $\mu = \mu_1$, respectively, with known variance σ^2. Make the form of the test as simple as you can.

   (b) Find the power $\beta(\mu)$ of the test in a) as function of μ.

   (c) For any given density functions p and q such that the Kullback Liebler Divergence

   $$D(p||q) = E_p\left[\log\frac{p(X)}{q(X)}\right] \quad \text{and the variance} \quad \tau^2_{p||q} = \mathrm{Var}_p\left[\log\frac{p(X)}{q(X)}\right]$$

   are finite, use the Central Limit Theorem to derive an approximation to the Neyman Pearson test for a given level $\alpha \in (0,1)$.

   (d) Assuming in addition that $D(q||p)$ and $\tau^2_{q||p}$ are finite, likewise approximate the power function of the test in (c), and show that it recovers the normal case in (b).

2. Throughout this problem let $a > 0$ be known constant. Let f be the density function

   $$f(x) \propto a - x, \quad \text{for} \quad 0 < x < a.$$

   Assuming the only random variables you have access to are i.i.d. $U(0,1)$, give the details of a way to simulate a random variable with distribution f.