

COMPLEX ANALYSIS

Spring 2019

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Find

$$\int_0^{\infty} \frac{\ln x}{1+x^4} dx.$$

2. Find the number of zeros of the polynomial $z^4 + 4z^2 + z + 1$ in the unit disk.

3. Describe all functions holomorphic in $\mathbb{D} = \{z : |z| < 1\}$ for which $f\left(\frac{1}{n}\right) = -\frac{1}{n^2}$, $n \in \mathbf{N}$.

4. Let f, g be entire functions such that $|f| \leq |g|$. Prove that $f = cg$ for some $c \in \mathbf{C}$.