MATH 505a **QUALIFYING EXAM** **January 8, 2019**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a new page and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway. For problems with multiple parts, if you cannot get an answer to one part, you might still get credit for other parts by assuming the correct answer to the part you could not solve. Be aware of the passage of time, so that you can attempt all three problems. When a problem asks you to find something, you are expected to simplify the answer as much as possible.

1. Suppose that each of 5 jobs is assigned at random to one of three servers A, B and C. [For example, one possible outcome would be that job 1 goes to server B, job 2 goes to server C, job 3 goes to server C, job 4 goes to server B and job 5 goes to server A. "At random" here means that there are 3^5 equally likely outcomes.]

(a) Find the probability that server C gets all 5 jobs.

(b) Let S be the number of servers that get exactly one job. Find $\mathbb{E}\, S$.

(c) Find the probability that no server gets more than 2 jobs.

(d) Take the same story, but with m in place of 5 for the number of jobs, and n in place of 3 for the number of servers. Find the variance of S, in terms of m and n.

2. (a) Suppose that X is Poisson with parameter λ. Find the characteristic function of X.

(b) Suppose that X_n is Poisson with parameter λ_n and that $\lambda_n \to \infty$ as $n \to \infty$. Show using characteristic functions that $(X_n - \lambda_n)/\sqrt{\lambda_n}$ converges in distribution, and describe the limiting distribution.

3. A stick of length 1 is broken into two pieces at a uniformly distributed random point.

(a) Find the expected length of the smaller piece.

(b) Find the expected value of the ratio of the smaller length over the larger.

(c) Suppose the larger piece is then broken at a random point, uniformly distributed over its length, independent of the first break point. There are then three pieces. Find the probability the longest of the three has length more than 1/2.