COMPLEX ANALYSIS GRADUATE EXAM

Spring 2018

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Show that there is no holomorphic function f in $\mathbb{D} = \{|z| < 1\}$ so that $|f(z_n)| \to \infty$ whenever $|z_n| \to 1$ ($z_n \in \mathbb{D}$).

2. Assume that f is analytic in the unit disk $\mathbb{D} = \{z : |z| < 1\}$. Prove that f is odd if and only if all the terms in the Taylor series for f at $z_0 = 0$ have only odd powers.

3. Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2 + 1}\, dx$$

4. Let $\mathbb{D} = \{|z| < 1\}$ be the open unit disk and $\bar{\mathbb{D}}$ its closure. Let $f : \mathbb{D} \to \mathbb{C}$ be analytic on $\mathbb{D}$ and continuous on $\bar{\mathbb{D}}$. Assume that f takes only real values on $\partial\bar{\mathbb{D}} = \{|z| = 1\}$. Prove that f is constant.