

MATH 505a GRADUATE EXAM

Spring 2016

Answer as many questions as you can. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. If you cannot do part (a) of a problem, you can still get credit for (b), (c) etc. by assuming the answer to (a). Start each problem on a fresh sheet of paper, and write on only one side of the paper.

- (1) A stick of length 1 is broken at a point uniformly distributed over its length.
 - (a) Find the mean and variance of the sum S of the squares of the lengths of the two pieces.
 - (b) Find the density function of the product M of the lengths of the two pieces. Note that $M \in [0, \frac{1}{4}]$.

- (2) There are two types of batteries in a bin. The life span of type i is an exponential random variable with mean $\mu_i, i = 1, 2$. The probability of type i battery to be chosen is p_i , with $p_1 + p_2 = 1$. Suppose a randomly chosen battery is still operating after t hours. What is the probability that it will still be operating after an additional s hours?

- (3) Fix positive integers $m \leq n$ with $n > 4$. Suppose m people sit at a circular table with n seats, with all $\binom{n}{m}$ seatings equally likely. A seat is called *isolated* if it is occupied and both adjacent seats are vacant. Find the mean and variance of the number of isolated seats.