**Algebra Exam February 2016**

Show your work. Be as clear as possible. Do all problems.

1. Let $R$ be a Noetherian commutative ring with 1 and $I \neq 0$ an ideal of $R$. Show that there exist finitely many nonzero prime ideals $P_i$ of $R$ (not necessarily distinct) so that $\Pi_i P_i \subset I$ (Hint: consider the set of ideals which are not of that form).

2. Describe all groups of order 130: show that every such group is isomorphic to a direct sum of dihedral and cyclic groups of suitable orders.

3. Let $f(x) = x^{12} + 2x^6 - 2x^3 + 2 \in \mathbb{Q}[x]$. Show $f(x)$ is irreducible. Let $K$ be the splitting field of $f(x)$ over $\mathbb{Q}$. Determine whether $\mathrm{Gal}(K/Q)$ is solvable.

4. Determine up to isomorphism the algebra structure of $\mathbb{C}[G]$ where $G = S_3$ is the symmetric group of degree 3 (Recall that $\mathbb{C}[G]$ is the group algebra of $G$ which has basis $G$ and the multiplication comes from the multiplication on $G$).

5. If $F$ is a field and $n > 1$ show that for any nonconstant $g \in F[x_1, \ldots, x_n]$ the ideal $gF[x_1, \ldots, x_n]$ is not a maximal ideal of $F[x_1, \ldots, x_n]$.

6. Let $F$ be a field and let $P$ be a submodule of $F[x]^n$. Suppose that the quotient module $M := F[x]^n/P$ is Artinian. Show that $M$ is finite dimensional over $F$.