MATH 507a QUALIFYING EXAM Friday, February 13, 2015. One hour and 50 minutes, starting at 4:30pm.

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1. Let $X_n,\ n \geq 1$, be independent random variables such that each X_n has Poisson distribution with mean n^r for some real number r. Determine the values of r for which

$$\lim_{N\to\infty} \frac{\sum_{n=1}^{N} X_n}{\sum_{n=1}^{N} n^r} = 1$$

with probability one.

2. Let $X_n,\ n \geq 1$, be a sequence of iid random variables with a continuous distribution function. For $m \geq 1$, let E_m be the event that a record occurs at moment m:

$$E_1 = \Omega,\ E_m = \{X_m > X_k,\ 1 \leq k < m\},\quad m \geq 2.$$

Show that
a) $\mathbb{P}(X_m = X_n) = 0$ for $m \neq n$;
b) $P(E_n) = 1/n$;
c) E_n and E_m are independent if $m \neq n$;
d) With probability one, infinitely many records occur.

3. Let $X_n,\ n \geq 1$, be iid random variables that are uniform on $(0, 2.5)$, and let $Y_n = \prod_{k=1}^{n} X_k$. True or false: $\lim_{n\to\infty} Y_n = 0$ with probability one. Explain your conclusion.