541a Qualifying Exam

Spring, 2006

Name:

1	
2	
total	

1. Let $X_1, \cdots, X_n$ be a random sample from a $N(\theta, 1)$ distribution, where $\theta \in \mathbf{R}$ is an unknown parameter.

a) Find the Fisher information $I(\theta)$.

b) Find a complete and sufficient statistic for θ.

c) Find an unbiased estimate of $P(X \leq 0)$.

d) Find the UMVUE (uniformly minimum variance unbiased estimator) of $P(X \leq 0)$.

2. Let $X_1, \ldots, X_n$ be i.i.d. with $P(X_i = 1) = 1 - P(X_i = 0) = p, p \in (0, 1)$.

a) Show that the average, $\bar{X}_n$, is the MLE of p

b) Find the mean μ_n and variance σ_n^2 of $\bar{X}_n$, and invoke the Central Limit Theorem to determine the asymptotic distribution of $\bar{X}_n$, properly centered and $\sqrt{n}$ scaled.

c) Find the MLE $\hat{\theta}$ of the log odds parameter,

$$\theta = \log\left(\frac{p}{1-p}\right)$$

and, under proper centering and $\sqrt{n}$ scaling, apply the Delta Method to find its non-trivial asymptotic distribution.

d) Find a variance stabilizing transformation for the estimate of $\hat{p}$ in a). (Recall that a variance stabilizing transformation for the estimator $\hat{\theta}$ is a function g such that the (properly centered and scaled) non-trivial asymptotic distribution of $g(\hat{\theta})$ does not depend on any unknown parameters.)