MATH 505a **QUALIFYING EXAM** **February 1, 2006**

Last Name: ____________________ **First Name:**____________________

ID#: ______________________ **Signature:**______________________

1. A *run* in a sequence of coin tosses is a maximal subsequence of consecutive tosses all having the same outcome; for example HHHTHHTTH has 5 runs. A biased coin, with $p = \mathbf{P}(\text{heads}) \in (0,1)$, is tossed n times. Write $q = 1 - p$. Let R_n be the number of runs in the first n tosses. Find exact formulas for

a) $\mu_n = \mathbf{E}R_n$ and

b) $\sigma_n^2 = \text{Var}(R_n)$.

HINT: pay careful attention to boundary effects-what happens at the start and end of the sequence of n tosses. Note that $\mu_1 = 1, \sigma_1^2 = 0$, and use this as a check on your answers. Note also that $\mathbf{P}(R_2 = 1) = p^2 + q^2$, $\mathbf{P}(R_2 = 2) = 2pq$, so $\mu + 2 = 1 + 2pq$.

c) For the special case $p = 1/2$, the distribution of $R_n - 1$ is very well known distribution (e.g. Binomial, Poisson, Hypergeometric, Geometric, etc) NAME the distribution AND its parameter(s).

2. Assume the vector $\mathbf{X} = (X_1, \ldots, X_N)$ has a multivariate normal distribution $N(\mu, \mathbf{V})$, where μ is the vector of expected values and $\mathbf{V}$ is the covariance matrix. Let $c_1, \ldots, c_N$ be constants.

Find the distribution of $Y = \sum_{i=1}^{N} c_i X_i$.

3. Let $S_n = X_1 + \cdots + X_n, n \leq 1$, be a random walk, where $EX_k = \mu$ and $Var(X_k) = \sigma^2$, $0 < \sigma^2 < \infty$.

(a) Find the covariance $Cov(S_n, S_m)$ and the correlation coefficient $\rho(S_n, S_m)$ of S_n and S_m, $m \neq n$.

(b) Assume $n > m$. Find $\lim_{n\to\infty} Cov(S_n, S_m)$ and $\lim_{n\to\infty} \rho(S_n, S_m)$. Does $\lim_{n\to\infty} Cov(S_n, S_m)$ depend on the distribution of the increments? Does $\lim_{n\to\infty} \rho(S_n, S_m)$ depend on the distribution of the increments?