COMPLEX ANALYSIS GRADUATE EXAM

Fall 2022

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Evaluate $\displaystyle\int_0^\infty \frac{\cos 8x}{x^2+1}\,dx$.

2. Does there exist a function $f(z)$ holomorphic in the unit disk $|z| < 1$ so that $\lim_{|z|\to 1} |f(z)| = \infty$?

3. Let $f(z)$ be holomorphic in the disk $|z| < 2$. Show that

$$\max_{|z|=1} \left| f(z) - \frac{1}{z} \right| \geq 1.$$

4. Map the region $\Omega = \{z \in \mathbb{C} : \Im(z) > 0\} \setminus \{1 + it : 0 < t \leq 1\}$ conformally to the upper half plane.