MATH 505a **QUALIFYING EXAM** **September 2020**

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a new page and write on only one side of the paper. For problems with multiple parts, if you cannot get an answer to one part, you might still get credit for other parts by assuming the correct answer to the part you could not solve. Be aware of the passage of time, so that you can attempt all four problems.

(1) Let X_n have binomial $B(n,p)$ distribution.

(a) Find $E\left(\frac{1}{X_n+1}\right)$. Simplify your answer so it does not involve a sum up to $n, n+1$, etc.

(b) Suppose $p = p_n$ and $np_n \to \lambda$ as $n \to \infty$, with $\lambda \in (0,\infty)$. Find $\lim_n E\left(\frac{1}{X_n+1}\right)$. Is it the same as $\lim_n \frac{1}{E(X_n+1)}$?

(2) Let X, Y be independent with $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ distribution.

(a) Find $P(X = k \mid X+Y = n)$ for $0 \le k \le n$. Simplify your answer so it does not involve a sum. Do the actual calculation, don't just cite a theorem.

(b) Find $E(X^2 + Y^2 \mid X+Y = n)$.

(3) The county hospital is located at the center of a square whose sides are 2 miles wide. If an accident occurs within this square, then the hospital sends out an ambulance. The road network is rectangular, so the travel distance from the hospital, at coordinates $(0,0)$, to the point (x,y) is $|x|+|y|$. If an accident occurs at a point that is uniformly distributed in the square, find the mean and variance of the travel distance of the ambulance.

(4) Let X be a finite set X, and let P and Q be probabilities on X. Define the total variation distance between P and Q by

$$||P-Q||_{TV} = \frac{1}{2}\sum_{x\in X}|P(x)-Q(x)|.$$

Prove that

$$||P-Q||_{TV} = \max_{A\subseteq X}|P(A)-Q(A)|,$$

where the maximum is over subsets A of X.