1. Suppose lifetimes $X_1, \ldots, X_n, X_{n+1}, \ldots, X_{n+m}$ of $n+m$ lightbulbs are independent and have the exponential distribution

$$p(x;\theta) = (1/\theta)\exp(-x/\theta)\mathbf{1}(x > 0)$$

with unknown parameter θ. The first n lifetimes have been observed precisely, but the only information recorded on the final m observations is whether or not the bulb lasted longer than some time t. We consider using the EM algorithm to compute the maximum likelihood estimate of θ.

   a. Write down the full likelihood function, that is, had all failure times been observed, and the full log likelihood.

   b. Write down the maximum likelihood estimate that is obtained by using the full likelihood.

   c. For X an exponential variable with the same distribution as the data, compute

$$E[X|X > t] \quad \text{and} \quad E[X|X < t].$$

   d. Describe the E and M step for computing the maximum likelihood estimate of θ under the data as observed.

2. Let $X_1, \ldots, X_n$ be i.i.d. $N(\mu, 1)$ random variables.

   (a) Find the likelihood ratio test of size $0 < \alpha < 1$ for testing $H_0 : \mu = 0$ against $H_a : \mu \neq 0$.

   (b) Is this test uniformly most powerful? Justify your answer.

   (c) Does a uniformly most powerful test exist for this problem? Either exhibit such a test or prove that none exists.