

COMPLEX ANALYSIS GRADUATE EXAM

Fall 2017

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Let $f(z) = u(z) + iv(z)$ be an entire function and assume that $|u(z)| \geq |v(z)| \forall z \in \mathbb{C}$. Show that f is a constant.

2. Let $\alpha \in (0, 1)$ and $n \in \mathbb{N}$. Prove that the equation $e^z(z-1)^n = \alpha$ has exactly n simple roots in the right half plane $\{z : \Re(z) > 0\}$.

3. Evaluate the integral

$$\int_0^{2\pi} \frac{dt}{\cos t - 2}.$$

4. Write an entire function f which has the simple zeroes $1, 4, 9, 16, 25, \dots$ and has no other zeroes.