MATH 505a **QUALIFYING EXAM** **August 2017.**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a new page and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway. For problems with multiple parts, if you cannot get an answer to one part, you might still get credit for other parts by assuming the correct answer to the part you could not solve.

When the problem asks you to compute something, you are expected to simplify the answer as much as possible (unless explicitly instructed otherwise).

1. Let X be uniform on $[1,5]$, let Y be uniform on $[0,1]$, and assume that X and Y are independent.

(a) Compute the probability density function of the product XY.

(b) Compute the cumulative distribution function of the ratio X/Y.

(c) Compute the characteristic function of the sum $X+Y$.

(d) Compute the moment generating function of the random variable $X - \ln(Y)$.

2. An urn contains $2n$ balls, coming in pairs: two balls are labeled "1", two balls are labeled "2",..., two balls are labeled "n". A sample of size n is taken without replacement. Denote by N the number of pairs in the sample. Compute the expected value and the variance of N. **You do not need to simplify the expression for the variance.**

3. Let $U_1, U_2, \ldots$ be iid random variables, uniformly distributed on $[0,1]$, and let N be a Poisson random variable with mean value equal to one. Assume that N is independent of $U_1, U_2, \ldots$ and define

$$Y = \begin{cases} 0, & \text{if } N = 0, \\ \max\limits_{1 \le i \le N} U_i, & \text{if } N > 0. \end{cases}$$

Compute the expected value of Y.