1. Let $X_1, \dots, X_n$ be a sample from distribution F, let $X_{(1)} \le \dots \le X_{(n)}$ be the corresponding order statistics, and let θ and $\tilde{\theta}$ be the population and sample median, respectively. Assume that the sample size is 3 ($n = 3$),

   (a) Find the distribution of the ordered bootstrap sample $(X^*_{(1)}, X^*_{(2)}, X^*_{(3)})$, where X^*_i's are randomly selected from the sample with replacement.

   (b) Determine the bootstrap estimator $\widehat{\lambda_1}$ of the bias of sample median, $\lambda_1 = E(\tilde{\theta}) - \theta$.

   (c) Determine the bootstrap estimator $\widehat{\lambda_2}$ of the variance of sample median, $\lambda_2 = Var(\tilde{\theta})$.

2. Denote $\mathbf{z} \in \mathbb{R}^2$ by $\mathbf{z} = (x, y)$, and let $\mathbf{Z}_1, \dots, \mathbf{Z}_n$ be independent with distribution $\mathcal{N}(0, \Sigma)$ where

$$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \quad \text{for } \rho \in (-1, 1), \text{ unknown.}$$

   a. Write down the $\mathcal{N}(0, \Sigma)$ density function, and the likelihood

$$L(\rho) = f(\mathbf{x}_1, \dots, \mathbf{x}_n; \rho)$$

   of the sample.

   b. Determine the Neyman Pearson procedure for testing $H_0 : \rho = 0$ versus $H_1 : \rho = \rho_0$ at level $\alpha \in (0, 1)$ for some $\rho_0 \neq 0$ in $(0, 1)$. (You do not need to explicitly write down any null distributions arising.)

   c. Determine if the test in b) is uniformly most powerful for testing $H_0 : \rho = 0$ versus $H_1 : \rho > 0$, and justify your conclusion.