COMPLEX ANALYSIS GRADUATE EXAM

Fall 2014

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Let $a > 1$. Compute

$$\int_0^\pi \frac{d\theta}{a+\cos\theta}$$

being careful to justify your methods.

2. Find the number of zeros, counting multiplicity, of $z^8 - z^3 + 10$ inside the first quadrant $\{z \in \mathbb{C} : \mathbb{R}\mathrm{e}\, z > 0, \mathbb{I}\mathrm{m}\, z > 0\}$.

3. Assume that $f(z)$ and $g(z)$ are holomorphic in a punctured neighborhood of $z_0 \in \mathbb{C}$. Prove that if f has an essential singularity at z_0 and g has a pole at z_0, then $f(z)g(z)$ has an essential singularity at z_0.

4. (i) Suppose that f is holomorphic on $\mathbb{C}$ and assume that the imaginary part of f is bounded. Prove that f is constant.
(ii) Suppose that f and g are holomorphic on $\mathbb{C}$ and that $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. Prove that there exists $\lambda \in \mathbb{C}$ such that $f = \lambda g$.