Spring 2018

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

Enumerative Geometry Beyond Numbers

Richard Thomas

Algebraic Geometry

It is an axiom of introductory accounts in algebraic geometry that they begin by describing the field as the study of zero loci of polynomial equations. This makes it sound really boring. In fact, it involves a fascinating interplay between two different disciplines, built on a duality,

$$\text{Geometry} \longleftrightarrow \text{Algebra}.$$

To a space we assign the functions on it,

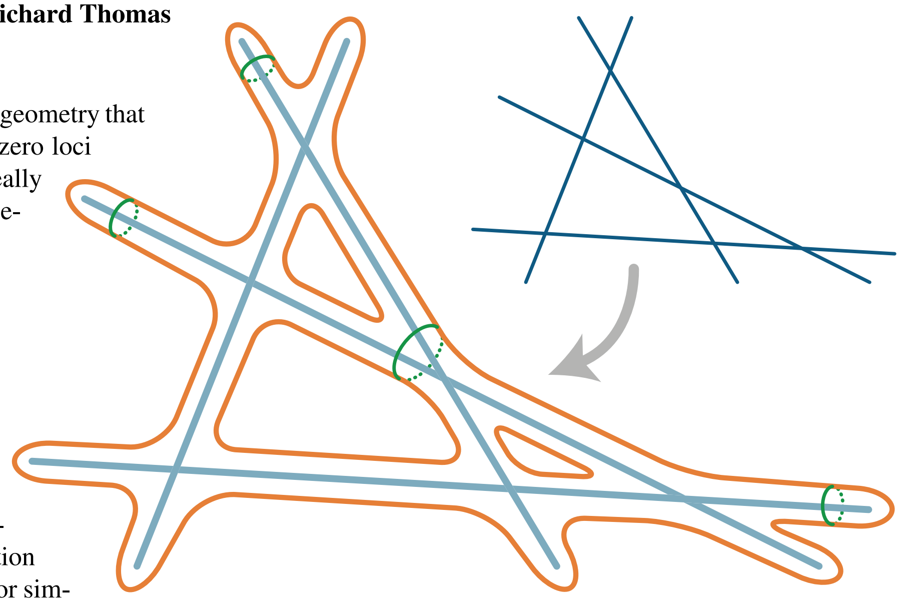
$$X \longmapsto \mathrm{Fun}(X) := \big\{\text{functions}\colon X \to \mathbb{C}\big\}.$$

We endow $\mathrm{Fun}(X)$ with the pointwise algebra structures of addition, multiplication, and multiplication by complex scalars. (We choose complex scalars for simplicity: Algebraic geometry over $\mathbb{R}$ is a more difficult subject with no fundamental theorem of algebra; other choices of scalars are even harder, and give us number theory.)

To go back, recovering the geometry X from the algebra $\mathrm{Fun}(X)$, we note that a point $x \in X$ gives a homomorphism to $\mathbb{C}$ by evaluating functions at x:

$$\mathrm{Fun}(X) \longrightarrow \mathbb{C}, \quad f \longmapsto f(x).$$

That is, evaluation at x commutes with addition and multiplication, and it turns out that this is *all* of the algebra homomorphisms.



Visualizing the geometry of the zero locus $p(x,y) = 0$ — the complex curve shown in orange. The intersection of four Riemann spheres (top right inset and light blue lines) is deformed to remove the singularities and yields a surface with three holes.

The type of functions we include in $\mathrm{Fun}(X)$ depends on what category of spaces we are working with. For instance, if X is a compact Hausdorff topological space, we take continuous complex-valued functions. The Gelfand–Naimark theorem then recovers X from the resulting C^*-algebra, with points of X corresponding to continuous homomorphisms which commute with complex conjugation.

(continued on page 8)

Making connections in mathematics is an integral part of what MSRI does. One example is the ongoing **Connections for Women** workshop series — this photo shows February's Group Representation Theory and Applications workshop. Elsewhere in this issue: Connections with the public in our **outreach highlights** (page 7); and connections with math's playfulness and challenges in the **Puzzles Column** (page 13).

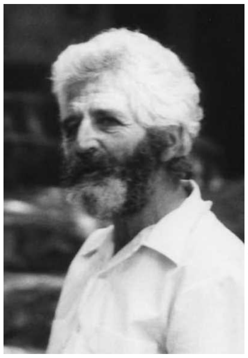
The View from MSRI

David Eisenbud, Director

Cup–pling Mathematicians

Rather than surveying MSRI, I want to write about a bit of MSRI history, one that has contributed to making MSRI a world center for collaborative mathematical research.

In the fall of 2002 I was on sabbatical from the MSRI directorship, enjoying a wonderful program on Commutative Algebra, and thus sitting, not in my director's office, but in a much higher place, a members office on the third floor. A knock on the door, and in walked an elderly stranger: Bill Craig.



John Kadvany / Wikimedia Commons / Public Domain

Readers of this column may now know more about the mathematician Walter Craig, currently at McMaster University and Bill's son, than about Bill Craig himself. At the time I met him, Bill was a short, imposing figure, a philosopher, logician and mathematician, a professor in the Logic Group in Berkeley since 1959. In 2002 he was 84, but the picture here shows him when he was 69.

Originated by Bill Craig and rescued by János Kollár, MSRI mugs strengthen research by making collaboration easier. The initial and second styles (top left and right) and the current design (below).

The Origin

Bill wanted something of me, and he wanted to give me something to make it possible: "I will donate \$30,000," he said, "to endow a Mug Fund at MSRI. You must agree to give every member a hand-thrown mug with his or her name handwritten on it in such a way that, when the owner holds it at tea, the name will be on the outside — thus different for right-handers or left-handers. In this way, no one will be embarrassed by not knowing the name of someone they want to talk to! — And mathematicians will take their mugs home and fondly remember a great time at MSRI."

An outsider might think such a gift frivolous, or at best a silly way to make lonely mathematicians happy. But Bill aimed very specifically at a different end: to strengthen research by making collaboration easier.

This was the very first time that someone had walked in to my office and offered a substantial gift, unsolicited, and of course I was surprised and charmed! I was also surprised by the specificity of this request, and thought maybe I could tweak it to make it easier to fulfill — but to no avail. Bill knew what he wanted to do, and would have no substitutions. There was one other specification: the gift was to be strictly anonymous: no one must know the benefactor. Only after Bill died, in 2016, did his son Walter mention the gift in public, which is why I now feel free to speak of it.

But back to 2002: It turned out, indeed, that one could purchase mugs to this specification, and the program began. I estimate that over the next years we made and gave away somewhere between 250 and 300 mugs each year, and they were much enjoyed by the members. The maker of the first design soon backed out (he wasn't making enough money), and we found the potter that designed the second wave. This design was in production until its maker passed away several years ago. But we managed to find another potter willing to do the job, and the design that we use now was born.

The Problem

Alas, there was a catch: At a 4% interest rate, the original donation was not enough to endow the program as Bill wished. He sadly agreed to let us spend down the principal to keep the program democratic, and to avoid having to limit the number of recipients. When the money ran out, Bill generously made a further, smaller gift, but warned us that this would be the last. Bill died, warmly remembered, in 2016. (Here is an example of a conference to honor him in 2007, when he was "only" 89: sophos.berkeley.edu/interpolations.) Once again the Mug Fund was in trouble.

The Rescue

In the spring of 2017, János Kollár was one of the winners of the Shaw Prize in Mathematics, and generously offered a large fraction of his prize to MSRI. We discussed how he would like his fund to be used, and I offered him several alternatives. Like Bill Craig, János (pictured here) strongly felt the value of strengthening social connections between mathematicians for the sake of encouraging research collaborations, and he chose — endowing the Mug Fund! The fund should now suffice for the foreseeable future: every member at MSRI can feel grateful to the generosity and understanding of mathematical ways shown by Bill Craig and János Kollár!

William Crow

Contents

The New Della Pietra Fellowship and . . .

In the Fall 2017 Emissary, MSRI announced that a donation by brothers Stephen and Vincent Della Pietra has made possible two new named fellowships at MSRI. A biography of Stephen Della Pietra accompanied the announcement. This spring, with the awarding of the second Della Pietra postdoctoral fellowship to Inna Entova-Aizenbud (also profiled here), we honor Vincent Della Pietra.

Vincent Della Pietra

Vincent Della Pietra received his bachelor's degree in physics from Princeton University in 1981, and his Ph.D. in mathematical physics from Harvard University in 1986. From 1987 to 1988, he was a postdoctoral fellow at The University of Texas at Austin. From 1988 to 1989, he conducted postdoctoral research at the Institute for Advanced Study in Princeton.

From 1989 to 1995, Vincent was a research staff member at the IBM Thomas J. Watson Research Center in Yorktown Heights and Hawthorne, NY. As a project leader of the natural language understanding group, his primary research focused on machine translation and natural language understanding. In 1995 he joined Renaissance Technologies, where he currently co-manages the General Research Group and works on statistical methods to model the stock market.

Vincent is co-founder of the Della Pietra Lecture Series at Stony Brook University. This series brings world-renowned scientists to the Simons Center for Geometry and Physics, and is intended to bring awareness of recent and impactful discoveries in physics and mathematics to high school, undergraduate, and graduate students.

. . . Vincent Della Pietra Fellow

Inna Entova-Aizenbud, a member of this semester's program in Group Representation Theory and Applications, is the first recipient of the new Vincent Della Pietra fellowship. Inna was born in Moscow but her family went to Israel when she was young.

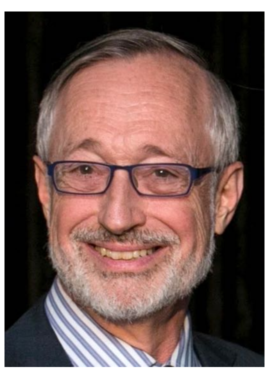
Inna Entova-Aizenbud

Inna got interested in mathematics early in her life, earning silver and bronze medals at the International Mathematics Olympiad for high school students as well as First Prize at the International Mathematics Competition for undergraduate students. Inna did her undegraduate study at Tel Aviv University and completed her Ph.D. at MIT with Pavel Etingof in 2015. Now Inna works at Ben Gurion University.

Inna's area of expertise is algebraic representation theory. She has obtained important results about Deligne tensor categories, the object generalizing symmetric, and general linear groups for nonintegral dimension. She has been able to give new conceptual proofs of previously known relations between Kronecker coefficients while discovering several new relations. And she has also constructed universal abelian tensor categories using representation theory of algebraic supergroups.

CME Group–MSRI Prize in Innovative Quantitative Applications

The twelfth annual CME Group–MSRI Prize in Innovative Quantitative Applications was awarded to Paul Milgrom, the Shirley and Leonard Ely professor of Humanities and Sciences in the Department of Economics, and professor, by courtesy, at both the Department of Management Science and Engineering and the Graduate School of Business at Stanford University.



Paul Milgrom

A leader in radio spectrum policy and auction theory and applications, Dr. Milgrom co-invented the major auction formats used for selling radio spectrum licenses in North America, Europe, Asia and Australia. Dr. Milgrom recently led the design of the $20 billion US Incentive Auction, which reallocated UHF-TV channels for use in mobile broadband. A panel discussion on "Frontiers of Research in Market Design" was held in conjunction with the award ceremony at a luncheon in Chicago on February 12, 2018.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical, or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the prize at tinyurl.com/cme-msri.

Directorate Lineup, 2018–19

During the academic year 2018–19, I'll have a little more time for mathematics, even while writing the NSF recompetition proposal (due in March 2019) and continuing to work on fundraising and board relations for MSRI. Our current Deputy Director, Hélène Barcelo, will be Acting Director, and Michael Singer will be Assistant Director (Michael was Deputy Director and then Acting Director during 2001–03). The Institute will be in good hands! I'll return to being Director, with Hélène as Deputy Director, in August 2019.

— David Eisenbud

Group Representation Theory and Applications

Gunter Malle

Groups are one of the most basic structures in mathematics and also play a role in many other sciences as the means to capture the essence of symmetry. Thus understanding groups and their properties is of fundamental importance. A natural form in which groups occur, and the best way to think about them, is via their representations — that is, their action on suitable mathematical objects. This could be by permuting a set, as for example the Galois group of a polynomial acts on its set of roots, or also by a linear action on a vector space, a so-called representation — that is, a group homomorphism

$$R : G \longrightarrow \mathrm{GL}(V)$$

from the group G to the group of invertible linear transformations of a vector space V.

Our program is devoted to the study of such representations. Two important classes of groups are finite groups and linear algebraic groups. In both cases, important aspects of their structure and representation theory are controlled by their composition factors, that is, by finite simple groups and simple algebraic groups, respectively. Improving the understanding of the basic (representation theoretic) properties of these two classes of groups and applying them in other areas is the main aim of our program.

Characters of Finite Groups

The proof of the classification of the finite simple groups (CFSG) has shown the importance of local methods: the subgroups of p-power order and their normalisers, the so-called p-local subgroups, control crucial aspects of the structure of a finite group G for any prime divisor p of its order. This type of local-global principle, strongly reminiscent of similar approaches in number theory, pervades modern finite group theory. Starting with the pioneering work of Richard Brauer in the 1950s, it has become more and more apparent recently that a similar connection should also hold for the representation theory of finite groups. This is the content of many fundamental conjectures in this area which are also studied in our program, some of them open since Brauer's times.

We illustrate this by discussing the most basic such conjecture. Let us recall that Georg Frobenius showed how to study representations of finite groups on complex vector spaces in terms of their trace functions, the so-called characters. He showed that a finite group G has only finitely many irreducible characters (corresponding to irreducible representations), usually denoted $\mathrm{Irr}(G)$. If $\chi \in \mathrm{Irr}(G)$ is a character, then its value on the identity element of the group is just the dimension of the vector space underlying a representation with character χ. One important goal is the understanding of this set $\{\chi(1) \mid \chi \in \mathrm{Irr}(G)\}$ of dimensions in some p-local way. It was more than 70 years after the invention of character theory that John McKay, in 1972, made the following astonishing observation on the characters of some of the newly discovered sporadic simple groups for the prime $p = 2$, which soon became a general conjecture:

Conjecture (McKay 1972)**.** *Let G be a finite group, p a prime, and P a Sylow p-subgroup of G with normaliser $N_G(P)$. Then*

$$|\mathrm{Irr}_{p'}(G)| = |\mathrm{Irr}_{p'}(N_G(P))|.$$

Here, $\mathrm{Irr}_{p'}(G)$ denotes the set of irreducible characters $\chi \in \mathrm{Irr}(G)$ of degree $\chi(1)$ not divisible by p. Thus, this quantity should be determined p-locally! In spite of various partial results, this easily stated fundamental conjecture remains open to the present day.

This is just the weakest of a whole hierarchy of local-global conjectures, most notably the Alperin–McKay conjecture, Brauer's height zero conjecture, and Dade's conjecture, which make predictions about properties of characters of a finite group in terms of its p-local structure. In a similar vein, Alperin's weight conjecture from 1986 claims that the number of non-projective irreducible representations of a finite group now over an algebraically closed field of positive characteristic p, up to isomorphism, is determined by p-local data. Some of these conjectures have been refined to also take into account additional structures, for example, the action of the absolute Galois group of $\mathbb{Q}$ on characters.

Finite Simple Groups

While these conjectures are open in general, a promising approach has emerged in the past ten years: work of M. Isaacs, G. Malle, G. Navarro, B. Späth, and P.H. Tiep has shown that all of these local-global, or counting, conjectures can be reduced to questions about quasi-simple groups. (A quasi-simple group is a finite group which modulo its centre is simple and which coincides with its commutator subgroup.) That is, if we can show that the representations of the finite quasi-simple groups satisfy certain (complicated) properties, then the conjectures hold for all finite groups. As all such groups are known by the CFSG, this may be a viable path to an eventual proof of these basic conjectures in finite group representation theory. Indeed, it has led to the 2016 proof of McKay's conjecture by B. Späth and the author for the situation in which it was originally stated, namely for the prime $p = 2$.

It has turned out, though, that our current knowledge on the representation theory of finite (quasi-)simple groups is not sufficient to verify the necessary conditions in full generality. Thus a further in-depth study of these is necessary. Results in this area are also expected to be applicable to a far wider range of questions, as earlier ones have already led to substantial progress in a vast number of applications in Lie theory, number theory, algebraic geometry, and combinatorics, to name a few.

The largest class of finite simple groups is formed by the simple groups of Lie type, which include for example the projective special linear groups $\mathrm{PSL}_n(\mathbb{F}_q)$, the projective symplectic groups $\mathrm{PSp}_{2n}(\mathbb{F}_q)$, or the groups of exceptional type $E_8(\mathbb{F}_q)$ over finite fields $\mathbb{F}_q$ (one infinite series for each choice of finite-dimensional

Focus on the Scientist: Robert Guralnick

Robert Guralnick is an outstanding mathematician who has contributed seminally to various important fields, including finite groups and finite simple groups, algebraic groups, representation theory, linear algebra, number theory, geometry, and combinatorics. He is one of the organizers of this semester's Group Representation Theory and Applications program.

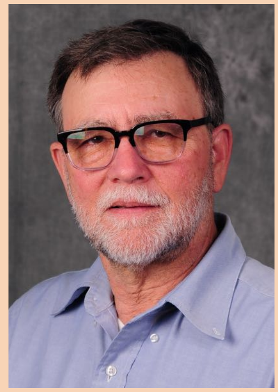
Michael Aschbacher introduced Guralnick at his JMM plenary talk in 2013 as the top person in the world in using group theory to solve problems in other areas of mathematics. Various highlights of Guralnick's work illustrate this.

Guralnick (with coauthors) vastly generalized a result of Zariski on maps from generic Riemann surfaces and their monodromy groups. He proved the Carlitz conjecture on exceptional polynomials, and Serre's conjecture, providing an analogue of Maschke's theorem in positive characteristic.

The notion of expander graphs is pivotal to computer science and communication networks. Guralnick took part in proving a spectacular result that random pairs of elements in various families of finite groups give rise to Cayley graphs that are expanders.

Studying generators for finite simple groups goes back to around 1900. Aschbacher and Guralnick showed that any finite simple group could be generated by two elements. In later work with other authors, Guralnick proved that one of them could be fixed. He has also been a leader in probabilistic generation of finite groups and analogues for algebraic groups.

Guralnick obtained many fundamental results on permutation groups (finite and algebraic). This includes a classification of finite primitive groups containing an element fixing at least half the elements, solving the Boston–Shalev conjecture about proportion of derangements, and getting very strong bounds for the base size of primitive actions.



Robert Guralnick

A fundamental problem in representation theory is the study of cohomology groups. Guralnick (with coauthors) proved remarkable results on the first and second cohomology groups, including a conjecture of Holt. This led to important collaborative results on short presentations of finite simple groups. Guralnick also solved conjectures of Peter Neumann from 1966 on fixed spaces of modules and contributed to the formidable inverse Galois problem.

Guralnick has received various honors in recognition of his groundbreaking work. For example, he was an invited ICM speaker in 2014 and was awarded the 2018 Frank Nelson Cole Prize in Algebra this past January. His sparkling thought and original interdisciplinary ideas greatly inspire many of us.

— Aner Shalev

simple complex Lie algebra and automorphism of its associated Coxeter diagram). Their complex irreducible characters were parametrized in the momentous work of George Lusztig, but many open questions still remain.

The key to a uniform treatment of these 16 infinite families of simple groups lies in the fact, as shown by R. Steinberg, that they may be constructed as the groups of fixed points under suitable Frobenius automorphisms of simple algebraic groups. The latter are amenable to the powerful methods from algebraic geometry, in particular the construction of representations on ℓ-adic or intersection cohomology groups. An important area of research consists in further exploiting this connection.

Algebraic Groups

This brings us to the realm of algebraic groups. The representation theory of (simple) algebraic groups defined over an algebraically closed field of characteristic zero, like the complex numbers, has been well understood since the middle of the last century. Not only is there a classification of all (finite-dimensional) irreducible representations in terms of combinatorial objects, so-called weights, but there are also formulas for their dimensions and finer invariants like weight spaces. The picture changes dramatically for groups defined over fields of positive characteristic: while the combinatorial classification of irreducible representations is still valid, there are no general results about their dimensions. In 1980, Lusztig proposed a conjecture predicting these dimensions under an explicit lower bound on the characteristic. By a result of Steinberg, his conjecture also implies a dimension formula for the absolutely irreducible representations of finite simple groups of Lie type in their defining characteristic.

Completing a program outlined by Lusztig, this conjecture was proved by H. Andersen, J. Jantzen, and W. Soergel in 1994 for large enough primes, but without giving any explicit lower bound. More recently P. Fiebig extended this result, obtaining an explicit (albeit enormous) bound on the characteristic above which the conjecture holds.

In 1990, Soergel proposed another approach to Lusztig's conjecture in terms of so-called Soergel bimodules, and later made a conjecture which would imply parts of Lusztig's conjecture. Although Soergel bimodules are explicit in principle, working with them is still quite complicated. Over the last five years, significant progress to overcome these difficulties was made. B. Elias and G. Williamson succeeded in giving a proof of Soergel's conjecture. The key to their proof is establishing the existence of certain deep Hodge theoretic structures on Soergel bimodules. One aim of our program is to uncover other situations in representation theory where Hodge theoretic structures are hiding.

Recently, Williamson produced many counterexamples to the original formulation of Lusztig's conjecture and showed that any bound on the characteristic will have to be exponential in the Coxeter number. Incidentally, Williamson's results also lead to counterexamples to a conjecture of G. James (1990) on the modular characters of the (finite) symmetric groups.

Hence, for "reasonable" primes we need a new conjecture to replace Lusztig's conjecture. It seems likely that this will use the p-canonical basis (defined using modular analogues of Soergel bimodules or parity sheaves). Establishing such a result would reduce several basic questions in modular representation theory to purely combinatorial calculations.

Categorical Aspects

It is believed that the counting conjectures described above are shadows of deep local-global categorical equivalences. While we do not as yet have the right general formulation, M. Broué's abelian defect group conjecture from 1988 proposes an explanation at least for groups G with abelian Sylow p-subgroups (or more generally for abelian defect groups): the numerical coincidences should come from an underlying derived equivalence between representation categories attached to G and to suitable local subgroups.

In their proof of this conjecture for the case of symmetric groups, J. Chuang and R. Rouquier introduced the notion of an $\mathfrak{sl}_2$-categorification, which since then has found its applications in areas as varied as Lie theory, geometric representation theory, representation theory of finite dimensional algebras, knot theory, and low-dimensional topology. This is an instance of what is nowadays called categorification, a quickly developing subject with applications all over mathematics and very tight and mutually beneficial connections to group representation theory. One basic example here is given by the modular representation theory of the symmetric groups and related Hecke algebras, which provides a categorification of highest weight representations of certain quantum affine Kac–Moody algebras.

The recent discovery by M. Khovanov, A. Lauda, and Rouquier of KLR algebras (or quiver Hecke algebras) can be considered as a far-reaching generalization of Chuang–Rouquier. Their representation theory has connections with the representation theory of symmetric groups and cyclotomic Hecke algebras discovered by J. Brundan and A. Kleshchev but also with canonical bases and cluster algebras.

Applications

The CFSG and the Aschbacher–Scott program on classifying maximal subgroups of finite simple groups have opened up an efficient way to approach problems in many areas of mathematics where group actions are involved, as in combinatorics, number theory, and algebraic geometry. Typically, one is led to establish a certain statement for groups possessing linear representations subject to some constraints. Using representation theory and the Aschbacher–Scott program, one reduces the initial problem to the case where the group in question is (close to being) simple (algebraic or finite). The CFSG and recent advances on representations of simple groups should enable one to identify the structure of a minimal counterexample (to the desired statement), which may then be ruled out by ad hoc methods using detailed knowledge of the representations of finite simple groups as obtained in the work described in the previous sections.

This strategy has proved successful in solving several longstanding problems, for example in number theory and algebraic geometry. Many of these recent results were obtained by members of the current program. The results concern monodromy groups of families of curves, character sums over finite fields, holonomy groups of vector bundles, quotients of Calabi–Yau varieties, crepant resolutions, Beauville surfaces, adequate subgroups and their connections to automorphy lifting theorems, the inverse Galois problem of realising groups as Galois groups, the proof of the Ore conjecture, and solutions of the Waring problem for finite simple groups, to name a few.

The interplay between group representation theory and other areas of mathematics has proved to be fruitful and beneficial for all involved parties. It is another primary goal of the current program to explore these interconnections further.

Forthcoming Workshops

Apr 9–13, 2018: *Representations of Finite and Algebraic Groups*

Apr 14–15, 2018: *The 2018 Infinite Possibilities Conference*

Aug 16–17, 2018: *Connections for Women: Hamiltonian Systems, from Topology to Applications through Analysis*

Aug 20–24, 2018: *Introductory Workshop: Hamiltonian Systems, from Topology to Applications through Analysis*

Oct 1–5, 2018: *Hot Topics: Shape and Structure of Materials*

Oct 8–12, 2018: *Hamiltonian Systems, from Topology to Applications through Analysis I*

October 10–11, 2018: *2018 Modern Math Workshop*

Nov 26–30, 2018: *Hamiltonian Systems, from Topology to Applications through Analysis II*

Summer Graduate Schools

Jun 11–22, 2018: *The ∂-Problem in the Twenty-First Century*

Jun 11–22, 2018: *Séminaire de Mathématiques Supérieures 2018: Derived Geometry and Higher Categorical Structures in Geometry and Physics*

June 17–30, 2018: *Mathematical Analysis of Behavior*

Jun 25–Jul 6, 2018: *Derived Categories*

Jun 25–Jul 6, 2018: *H-Principle*

Jul 1–21, 2018: *IAS/PCMI 2018: Harmonic Analysis*

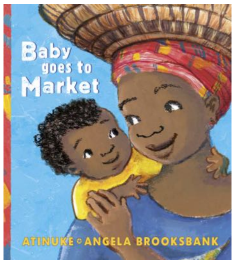
Jul 9–20, 2018: *Representations of High Dimensional Data*

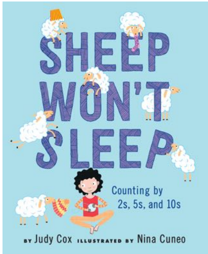
Jul 23–Aug 3, 2018: *From Symplectic Geometry to Chaos*

To find more information about any of these workshops or summer schools, as well as a full list of all upcoming workshops and programs, please visit msri.org/scientific.

Public Understanding of Mathematics — Highlights

2018 Mathical Book Prize Winners





MSRI's Mathical Book Prize recognizes outstanding fiction and literary nonfiction for youth aged 2–18. "We are trying to engage kids in the power and beauty of mathematics — power in that we want to give them confidence to consider and solve arithmetic, spatial, logical, structural, algebraic problems; beauty in that math can be employed to better understand and appreciate the wonders of nature and also human artistic expression, for example in fine art and music," said MSRI Board Chair Roger Strauch.

The prize, now in its fourth year, is selected annually by a committee of pre-K–12 teachers, librarians, mathematicians, early childhood experts, and others. Here are this year's winners:

Pre-K: *Baby Goes to Market* by Atinuke (Candlewick Press)

Grades K–2: *Sheep Won't Sleep: Counting by 2s, 5s, and 10s* by Judy Cox (Holiday House)

Grades 3–5: *A Hundred Billion Trillion Stars* by Seth Fishman (HarperCollins Children's Books)

The Mathical Book Prize is awarded by MSRI in partnership with the National Council of Teachers of English and the National Council of Teachers of Mathematics, and in coordination with the Children's Book Council. The Mathical list is intended as a resource for educators, parents, librarians, children, and teens.

Public Events

Here are some highlights of outreach events hosted by MSRI in 2017–18:

• A presentation for visiting researchers by mathematician-turned-journalist Erica Klarreich about **what makes a good math news story** and **the state of math journalism**. (Erica participated in MSRI's Journalist in Residence program in 2002 and has written for Quanta, Nature, Wired.com, Science News, and more.)

• Free **Harmonic Series** concerts featuring flutist Robert Stallman accompanied by Dmitriy Cogan on piano; performances by Berkeley residents Minsky Duo; and coming in May, a special **Math Lovers Forum** concert at the **Asian Art Museum** in San Francisco, featuring the mathematical, musical explorations of Purna Lohka Quartet.

• The annual **Celebration of Mind** festival of mathematical magic and games celebrating Martin Gardner's legacy, including a special workshop by Gardner's son James and magician Mark Mitton ...

• ... and Mitton also presented a public lecture at the **Commonwealth Club of San Francisco** on "Math, Magic, and Surprise" in April.

The **National Math Festival** returns to Washington, DC, on Saturday, May 4, 2019, at the Walter E. Washington Convention Center. Learn more at nationalmathfestival.org or sign up for the Festival newsletter at tinyurl.com/NMFnews.

• An online **Reddit Q&A** forum with former NFL player and mathematician John Urschel (Massachusetts Institute of Technology).

• Several events in conjunction with the **San Diego Festival of Science and Engineering**, with mathematical biologist Mariel Vazquez (UC Davis), technologist and space origami engineer Manan Arya (NASA's JPL), and a math collaboration adventure for 8–12th grade girls with Mia Minnes (UC San Diego).

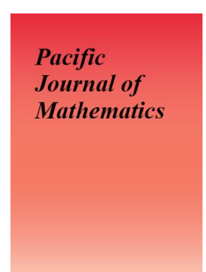
AMS/MSRI Congressional Briefing Report

On Dec 6, 2017, the American Mathematical Society and MSRI hosted a joint congressional briefing featuring Shafi Goldwasser, Director of the Simons Institute for the Theory of Computing at the University of California, Berkeley. Dr. Goldwasser (pictured) spoke to the Capitol Hill audience on "Cryptography: How to Enable Privacy in a Data-Driven World."

Additional speakers at the congressional briefing included House Minority Leader Nancy Pelosi, Rep. Jerry McNerney of California, and Rep. Dan Lipinski of Illinois. The next Congressional briefing will take place in May 2018, featuring Erik Demaine (Massachusetts Institute of Technology). You can learn more and view short films about past events: msri.org/congress.

Pacific Journal of Mathematics

*Founded in 1951, **The Pacific Journal of Mathematics** has published mathematics research for more than 60 years. PJM is run by mathematicians from the Pacific Rim and aims to publish high-quality articles in all branches of mathematics, at low cost to libraries and individuals. PJM publishes 12 issues per year. Please consider submitting articles to the Pacific Journal of Mathematics. The process is easy and responses are timely. See* msp.org/publications/journals/#pjm.



Enumerative Geometry Beyond Numbers

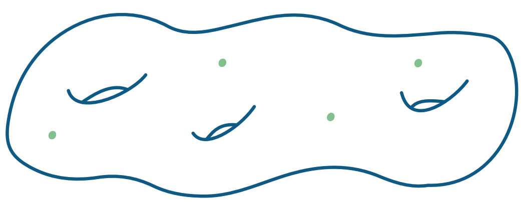
(continued from page 1)

The most familiar setting is linear algebra, where X would be a (finite-dimensional, complex) vector space and we take only linear functions. Therefore $\mathrm{Fun}(X)$ is the dual vector space X^*, and there is no multiplication structure. The homomorphisms $X^* \to \mathbb{C}$ are just linear functionals, recovering $X^{**} = X$.

Allowing multiplication of these linear functionals gives polynomial functions, and this corresponds to (affine) algebraic geometry. So, for instance, X might be the zero locus of some polynomials $p_i(x_1, \ldots, x_n)$ on $\mathbb{C}^n$,

$$\mathrm{Fun}(X) \;=\; \mathbb{C}[x_1, \ldots, x_n]/(p_1, \ldots, p_k),$$

and any algebra homomorphism ϕ from this algebra to $\mathbb{C}$ is evaluation at a point $x \in \mathbb{C}^n$ with $p_i(x) = 0\ \forall i$. In fact, ϕ is determined by its value on the generators $x_1, \ldots, x_n$; setting $a_i = \phi(x_i)$, we find ϕ is evaluation at $a := (a_i)_{i=1}^n \in \mathbb{C}^n$, and the well-definedness of ϕ on the quotient $\mathrm{Fun}(X)$ above forces $p_i(a) = 0\ \forall i$.



The geometry of the zero locus of a degree 4 polynomial in 2 variables $\{(x,y) \in \mathbb{C}^2 \colon p(x,y) = 0\}$ turns out to be a surface with three holes and four punctures at infinity.

To exploit the geometry $\longleftrightarrow$ algebra duality, we hope that what looks complicated from one point of view might be simple from the other. (Of course life often isn't like that, and what looks hard from one point of view looks harder from the other. But using a combination of both perspectives usually makes progress easier.)

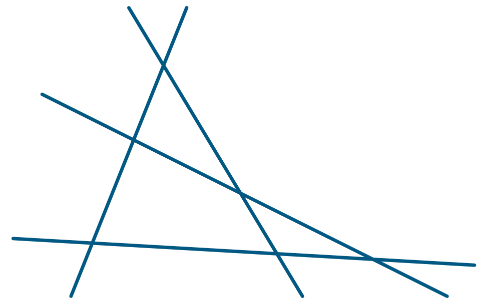
For instance, a degree 4 polynomial in two variables

$$p(x,y) \;=\; 7x^4 - 3x^3y + \ldots$$

is surely a simple bit of algebra, but the geometry of its zero locus is nontrivial: it is hard at first to see that it is a surface with three holes (and four punctures at infinity), as shown in the figure above.

One (geometric) way to see this is to change the coefficients of the polynomial until it is a product of four linear factors: $p_0(x,y) = \prod_{i=1}^4 (a_i x - b_i y)$. Then the zero locus will be a union of four complex lines $\mathbb{C} \subset \mathbb{C}^2$. Considering each to be a Riemann sphere (punctured once), we get four spheres which intersect in a pattern of the form shown in the figure at the top of the next column.

Near any intersection point we can choose coordinates so that the equation $p_0 = 0$ looks to leading order like $xy = 0$. As we start to deform p_0 back towards p, this equation deforms to $xy = \epsilon$, a



One way to visualize the geometry is to consider the union of four (real) lines $\{(x,y) \in \mathbb{R}^2 \,:\, p_0(x,y) = 0\}$. Over the complex numbers this becomes four Riemann spheres.

cylinder that replaces the intersection point with a small neck, as in the figure at the bottom of the page.

The upshot is the picture that appears at the beginning of the article on the cover, in which we can see the three holes. We leave it as a combinatorial exercise for the reader to show similarly that a generic degree d polynomial p has zero locus a surface with $(d-1)(d-2)/2$ holes (and d punctures).

Enumerative Algebraic Geometry

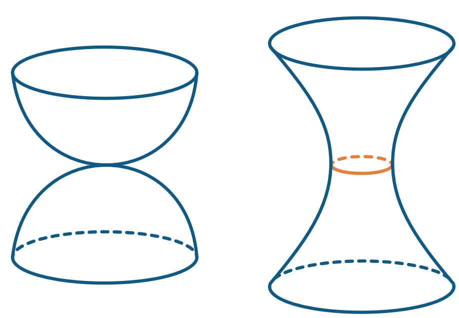
These surfaces are called complex algebraic curves, since they have one complex dimension. A classical question of enumerative algebraic geometry is how many curves exist in some higher dimensional algebraic variety, satisfying some conditions.

For instance, Euclid told us that there is a single line (degree 1 curve) between any two points in the plane. This continues to be true for complex lines through two complex points of $\mathbb{C}^2$.

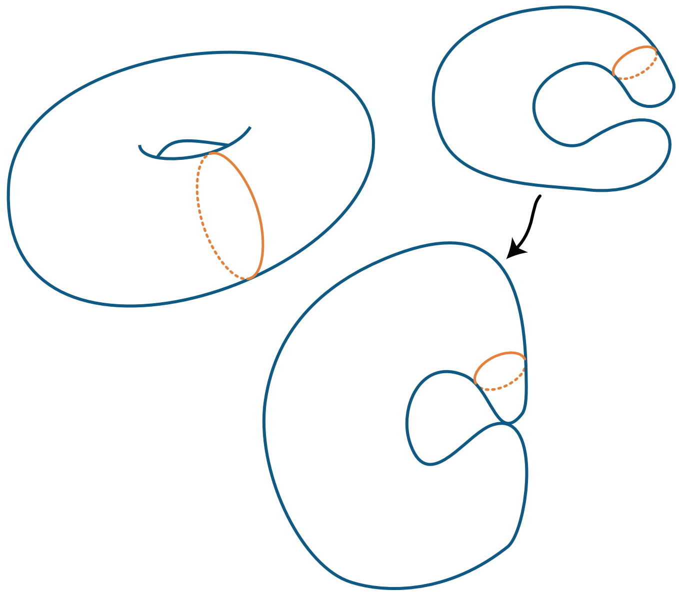
Apollonius showed there is a single conic (degree 2 curve) through five generic points in the plane. Already here it is best to use complex numbers, so the conic actually has some points (for instance $x^2 + y^2 + 1 = 0$ has no real points).

Cubic curves have one hole (that is, genus $(d-1)(d-2)/2 = 1$ when $d = 3$) but for some choices of coefficients they become singular curves of genus 0. (See the figure on the next page.)

Counting genus 0 curves turns out to be easier than counting arbitrary curves, and around 1850 Chasles showed the number of genus 0 cubics through eight generic points is 12. In 1873 Zeuthen showed that the number of genus 0 quartics through 11 generic



Near the intersection points, $p_0 = 0$ is given by $xy = 0$ to leading order. As we deform p_0 back toward p, the equation deforms to $xy = \epsilon$ and each singularity is replaced with a small neck.



For some choices of coefficients, a smooth cubic of genus 1 (far left) can become a singular, genus 0 cubic (bottom right) — the image of a sphere (top right).

points is 620. But each calculation was much harder than the previous, and no pattern was emerging. There, more or less, the field remained for more than a century until new ideas came into the subject from symplectic geometry and string theory.

This led to the introduction of Gromov–Witten invariants counting holomorphic (or algebraic) maps from a complex algebraic curve (or a Riemann surface) into a fixed target variety. Even though algebraic geometry is a rigid subject without many perturbations, many of the quantities of interest are inherently topological, and the setup of Gromov–Witten (GW) theory reflects that. It introduced a lot more flexibility into enumerative problems, for instance allowing us to remove conditions like "generic" from the above statements. This led to a completely different philosophy and point of view on enumerative problems. Using their added flexibility, Kontsevich almost immediately calculated *all* the genus 0 GW invariants of the plane, incorporating the classical statements above into a single recursion.

Theorem (Kontsevich 1994). *The number N_d of genus 0 curves of degree d through $3d-1$ points in the plane is determined by the recursion*

$$N_d = \sum_{i+j=d} N_i N_j i^2 j \left[j\binom{3d-4}{3i-2} - i\binom{3d-4}{3i-1} \right].$$

For $d = 1, 2, \dots$, this gives $N_d = 1, 1, 12, 620, 87\,304, 26\,312\,976, 14\,616\,808\,192, 13\,525\,751\,027\,392, \dots$

Higher genus GW theory — counting holomorphic maps of complex curves with holes into a fixed target — turns out to be *much* harder. In fact, counting the number of curves in a point required four Fields medallists (Witten, Kontsevich, Okounkov, Mirzakhani) and a Clay Research Award winner (Pandharipande). (The ridiculous-sounding GW theory of a point actually has content. It "counts" abstract curves (with their unique map to the point), and in higher genus there are whole parameter spaces of these curves (or equivalently, of different complex structures on the fixed underlying topological manifold) over which we compute natural integrals to do the counting.) In 2003 Okounkov and Pandharipande solved the GW theory of curves. There has been some progress on the GW theory of complex surfaces via Taubes's work on Seiberg–Witten theory.

But the case we (and string theorists) really care about is complex dimension 3. Here progress came from mathematics in the form of the MNOP conjecture of Maulik–Nekrasov–Okounkov–Pandharipande.

At one level, this concerns two different ways to describe holomorphic curves in a variety X, as either holomorphic maps

$$\text{parameterized curves } f\colon C \to X,$$

or as the zeros of holomorphic functions

$$\text{unparameterized curves } C = \{f_i = 0\} \subset X.$$

So for instance a conic $x^2 + ay = 0$ in the plane is an unparametrized curve which we can see as parametrized by $\mathbb{C}$ by the holomorphic map

$$\mathbb{C} \longrightarrow \mathbb{C}^2, \quad t \longmapsto (ia^{1/2}t, t^2).$$

Thinking of curves via maps leads naturally to a certain compactification of the moduli space (or parameter space) of all curves in X. Thinking of curves via the equations which cut them out, or more properly via their ideal sheaves, leads naturally to a very different compactification. For instance, we can take $a \to 0$ in the example conic above. The parametrization $\mathbb{C} \longrightarrow \mathbb{C}^2$ tends to a 2:1 cover

$$t \longmapsto (0, t^2)$$

of the line $x = 0$, whereas the equation $x^2 + ay = 0$ tends to $x^2 = 0$ (which scheme theory allows us to think of as a thickened line — the doubled y-axis).

These differences at the boundary of the moduli spaces make huge differences to the invariants one gets by integrating over them. The first compactification leads to GW invariants, the second to stable pair invariants. The MNOP conjecture is the very reasonable and understandable statement

$$\text{GW} = \text{stable pairs},$$

but with "=" a ridiculously complicated (near incomprehensible) binary relation:

$$\sum_g \text{GW}_g\, u^{2g-2} = \sum_n P_n\, q^n, \quad q = -e^{iu}.$$

Here we fix a target variety X, a degree (or "homology class") of the curves we count, and the same incidence conditions on both sides. Then GW_g counts holomorphic maps of curves of genus g, while P_n counts stable pairs with holomorphic Euler characteristic n (when the curve is not thickened, this amounts to $n = 1 - g$).

The conjecture is that the right hand side is the Laurent series of a rational function of q which is invariant under $q \leftrightarrow q^{-1}$, which means the change of variables $q = -e^{iu}$ makes sense and gives real coefficients. (For instance, the series $q - 2q^2 + 3q^3 - \dots$ is not $q \leftrightarrow q^{-1}$ invariant, but sums to a rational function $q/(1+q)^2$,

Focus on the Scientist: Cristina Manolache

Cristina Manolache is a member of this semester's program on Enumerative Geometry Beyond Numbers. Cristina began her studies at the University of Bucharest, where she completed a bachelor's degree in mathematics. She then enrolled in the English language master's program in mathematics at the University of Kaiserslautern, which she completed under the direction of Andreas Gathmann.

Cristina Manolache

She then joined the Ph.D. program in geometry at the International School of Advanced Studies (SISSA), where I met her. It was immediately clear that Cristina was an unusually brilliant student; in fact, in fifteen years of getting to work with four to six students per year, I cannot remember anyone with the same facility for mathematics.

It's not correct to say Cristina understood difficult proofs; rather, she seemed to absorb them, digest them, and reformulate so that they appeared natural. She (and I as her advisor) were lucky in that she met Ionuț Ciocan-Fontanine (also a member of this semester's program at MSRI) when he visited SISSA; she later visited him in Minnesota, and he acted as her co-advisor, leading her to complete an elegant proof of a long sought result (the functionality of virtual pullbacks) well within the third year of her Ph.D. studies.

When shortly thereafter Angelo Vistoli, one of the world leaders in the field, visited SISSA, I suggested she give a talk on her theorem. In the talk, she also presented a new theorem, whose proof wasn't yet completely written down. Vistoli said the theorem seemed wrong to him, and that he thought he might have a counterexample. She answered calmly that he couldn't have one, because she had proved the theorem. The next morning Vistoli confirmed that she was right.

With such potential it wasn't difficult for her to secure a string of top level positions, first in Berlin to work in the group of Gavril Farkas, then at Imperial College London, where she first had a Marie Curie grant and is now a Royal Society Dorothy Hodgkin Fellow in the Geometry Group of the Department of Mathematics.

Besides continuing her research work in algebraic geometry, she has also started advising Ph.D. students. One of them is attending this semester's program at MSRI, and together with another student who has taken part in the program's workshop, the pair will graduate in a few months, each with a number of papers.

Cristina has retained, and if possible deepened, her ability to simplify the most difficult mathematical arguments. Last year, together with Melissa Liu, she was lecturer at a summer school at SISSA where she managed to explain spaces of quasi maps to Ph.D. students, from the initial motivation through the very technical definition to the enumerative applications. We expect only more and better work from her in the future.

— Barbara Fantechi

which *is*.) We then analytically continue and expand around $u = 0$, that is, $q = -1$, to get the left hand side. In particular, to determine one GW invariant via the MNOP conjecture, one needs to know many stable pair invariants, and vice-versa.

Building on work of Maulik–Oblomkov–Okounkov–Pandharipande for targets X with toric symmetry, Pandharipande and Pixton have now proved the MNOP conjecture for many geometries X (including most Calabi–Yau threefolds). This has led to a flurry of progress in computing higher genus GW invariants of targets, exploiting the fact that stable pair theory is more linear and so slightly easier to calculate.

Beyond Numbers

Enumerative geometry has always been about more than just numbers. From the beginning, the numbers were packaged into connections, Frobenius structures, or generating series that then turned out to define special functions like modular forms or τ-functions for integrable hierarchies. As such they had links with mirror symmetry, representation theory, integrable systems, and even number theory.

Other, more complicated, structures like topological quantum field theories and geometric actions of quantum groups are now emerging from enumerative geometry, relating it to geometric representation theory, string theory, knot theory, and other topics. For instance, Maulik–Okounkov have given geometric constructions of elements of Yangians (infinite dimensional analogues of quantum groups) from curve counting moduli spaces. And many people have related knot and link invariants to GW invariants.

Enumerative invariants themselves are also being refined and categorified. Kass–Wickelgren and Levine have lifted counts to the Grothendieck–Witt ring. Joyce–Song and Kontsevich–Soibelman have shown how to work with something like the whole cohomology of the moduli space of curves in a target, instead of specific integrals over it. Instead of numbers they produce polynomials, or graded vector spaces, from which the original numbers can be recovered — by evaluating the polynomial at 1, or taking the (super)dimension of the vector space.

Early calculations of Behrend–Bryan–Szendrői and others show that generating series of refined invariants also give interesting special functions. In some cases the refinement replaces integers n by their quantum analogues

$$[n]_q := \frac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}} \xrightarrow[q \to 1]{} n.$$

They give geometric interpretations of various quantum formulae. Other refinements, calculations, and new structures in enumerative geometry are being found in the MSRI program as you read this!

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 15**, **October 15**, and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 15**, **October 15**, or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 15**, **October 15** or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

Call for Membership

MSRI invites membership applications for the 2019–2020 academic year in these positions:

Research Professors	by October 1, 2018
Research Members	by December 1, 2018
Postdoctoral Fellows	by December 1, 2018

In the academic year 2019–2020, the research programs are:

Holomorphic Differentials in Mathematics and Physics
Aug 12 – Dec 13, 2019
Organized by Jayadev Athreya, Steven Bradlow, Sergei Gukov, Andrew Neitzke, Anna Wienhard, Anton Zorich

Microlocal Analysis
Aug 12 – Dec 13, 2019
Organized by Pierre Albin, Nalini Anantharaman, Kiril Datchev, Raluca Felea, Colin Guillarmou, Andras Vasy

Quantum Symmetries
Jan 21 – May 29, 2020
Organized by Vaughan Jones, Scott Morrison, Victor Ostrik, Emily Peters, Eric Rowell, Noah Snyder, Chelsea Walton

Higher Categories and Categorification
Jan 21 – May 29, 2020
Organized by David Ayala, Clark Barwick, David Nadler, Emily Riehl, Marcy Robertson, Peter Teichner, Dominic Verity

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org after August 1, 2018. For more information about any of the programs, please see msri.org/scientific/programs.

Named Positions, Spring 2018

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chern, Eisenbud, and Simons Professors

Group Representation Theory and Applications
Radha Kessar, City University London
Alexander Kleshchev, University of Oregon
Martin Liebeck, Imperial College
Raphael Rouquier, University of California, Los Angeles
Pham Tiep, University of Arizona

Enumerative Geometry Beyond Numbers
Kai Behrend, University of British Columbia
Tom Bridgeland, University of Sheffield
Jim Bryan, University of British Columbia
Tobias Ekholm, Uppsala University
Sheldon Katz, University of Illinois at Urbana-Champaign
Chiu-Chu Melissa Liu, Columbia University
Dusa McDuff, Barnard College, Columbia University

Named Postdoctoral Fellows

Group Representation Theory and Applications
Della Pietra: Inna Entova-Aizenbud, Ben-Gurion University
Strauch: Shotaro Makisumi, Stanford University
Uhlenbeck: Robert Muth, Tarleton State University

Enumerative Geometry Beyond Numbers
Viterbi: Yaim Cooper, Harvard University
McDuff: Jørgen Vold Rennemo, University of Oslo

Clay Senior Scholars

The Clay Mathematics Institute (www.claymath.org) has announced the 2018–19 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2018–19.

Hamiltonian systems, from Topology to Applications through Analysis (Fall 2018)
Albert Fathi, Georgia Institute of Technology

Derived Algebraic Geometry (Spring 2019)
Dennis Gaitsgory, Harvard University

Birational Geometry and Moduli Spaces (Spring 2019)
Claire Voisin, Collège de France

Named Postdocs — Spring 2018

McDuff

Jørgen Vold Rennemo

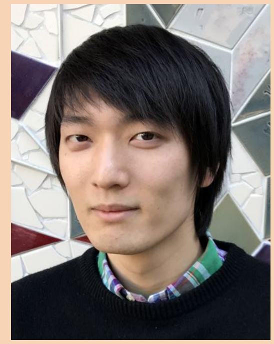
Jørgen Vold Rennemo is the McDuff Postdoctoral Fellow for the Enumerative Geometry Beyond Numbers program. His research interests include the derived category of coherent sheaves (in particular the application of homological projective duality to classical geometry questions) and the enumerative geometry of Calabi–Yau threefolds. A native of Lillehammer, Jørgen is a four time winner of the Norwegian Math Olympiad and was at one point a juggler in a circus. He earned his bachelor's and master's degrees from the University of Oslo, and his Ph.D. at Imperial College London, under the supervision of Richard Thomas. Although he uses very modern techniques, Jørgen's research has applications to very classical questions of enumerative geometry. For example, in joint work with Sjoerd Beentjes and John Calabrese, he recently made an important advancement in in Donaldson–Thomas theory by proving the crepant resolution conjecture with a proof that cleverly wields new techniques for wall-crossing in the derived category. *The McDuff fellowship was established by an anonymous donor in honor of Dusa McDuff. She is an internationally renowned mathematician, a member of the National Academy of Sciences, and a recipient of the AMS Leroy P. Steele Prize (2017). She is also currently a trustee of MSRI.*

Uhlenbeck

Robert Muth

Robert Muth is the recipient of the Uhlenbeck postdoctoral fellowship this spring. He is a member of the Group Representation Theory and Applications program. Robert completed his Ph.D. in 2016 from the University of Oregon under the supervision of Alexander Kleshchev and is currently an Assistant Professor at Tarleton State University in Stephenville, Texas. Robert's graduate research focused on the representation theory of Khovanov–Lauda–Rouquier algebras of affine type, especially affine type A. Currently, he is interested in superalgebra categorification and combinatorics. He has developed a new approach to super Robinson–Schensted–Knuth correspondence with symmetry. A major ongoing project, joint with Kleshchev, is "Schurification of superalgebras," generalizing a construction of Turner's which describes RoCK blocks of symmetric groups up to Morita equivalence. *The Uhlenbeck fellowship was established by an anonymous donor in honor of Karen Uhlenbeck, a distinguished mathematician and former MSRI trustee. She was the first female mathematician named to the National Academy of Sciences (1986) and is a recipient of the AMS Leroy P. Steele Prize and a MacArthur "Genius" Fellowship.*

Strauch



Shotaro Makisumi

Shotaro (Macky) Makisumi is the Strauch Postdoctoral Fellow in the Group Representation Theory and Applications program. Shotaro has earned his undergraduate degree (summa cum laude) in 2012 from Princeton University where he was advised by Peter Sarnak; at that time he received the Covington Prize awarded for excellence to two graduating seniors. Shotaro obtained his Ph.D. from Stanford University in 2017 under the direction of Zhiwei Yun. He now holds the position of Ritt Assistant Professor at Columbia University. Shotaro works in representation theory with an emphasis on geometric, combinatorial, and diagrammatic methods. He is an expert in geometric methods in modular representation theory, a rapidly developing area at the forefront of today's representation theory. In particular, Makisumi (in work partly joint with Achar, Riche, and Williamson) has used new, sophisticated ideas to develop methods originally coming from algebraic geometry to apply them to the intricate setting of a positive characteristic base field. *The Strauch Fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees.*

Viterbi

Yaim Cooper is the Viterbi Postdoctoral Fellow in this semester's Enumerative Geometry Beyond Numbers program. Yaim earned her Ph.D. under the supervision of Rahul Pandharipande. Her research focuses on the study of moduli spaces of curves in algebraic varieties. One focus of her work is the so-called Severi problem of counting curves of given homology class and genus (or, equivalently, number of nodes, the simplest singularities) on a smooth projective surface. Using ideas coming from physics to shift away from computing one invariant at a time and towards collecting them together as part of a larger geometric structure, Yaim (partially in join work with her advisor Rahul Pandharipande) has done work that packages all the enumerative geometry of curves on a minimal rational surface into a single operator on a Fock space. *The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.*

***Inna Entova-Aizenbud**, a postdoc in GRTA and the recipient of the new Vincent Della Pietra Fellowship, is* *profiled on page 3.*

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

MSRI has supported the Bay Area Mathematical Olympiad (BAMO) from its inception, and this year we had the biggest crowd ever, filling the Simons Auditorium to capacity! The 20th BAMO exam included some especially interesting problems, four of which appear here as problems 3–6.

1. Let n be a positive integer whose base 10 expansion is a nondecreasing sequence of digits. In addition, assume that the ones digit is strictly bigger than the tens digit. Prove that the digit sum of $9n$ is 9.

Comment: This is from Felix Lazebnik, "Surprises, Surprises, Surprises," *Mathematics Magazine* (June 2014).

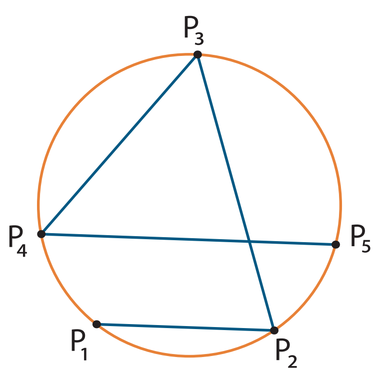
2. Find the smallest of number of "control weights," each weighing a nonintegral number of grams, which can balance all integer weights from 1 g to 40 g on a pan balance when the integer weight is on the right pan, and a suitable subset of control weights is on the left.

Comment: From our intrepid problem composer, Gregory Galperin.

3. A square of side 1 can be cut into four rectangles R_i in many ways. Describe the set of all possible values of the sum $\sum \mathrm{perim}(R_i)$ of the perimeters of those four rectangles. (If you wish, generalize.)

Comment: The final parenthetical question was not actually included on BAMO.

4. Let $x_1, \ldots, x_{2018}$ be a sequence of elements of $\{-1, 1\}$. Let $S = \sum x_i x_{i+1}$ be the sum of all of the products of adjacent numbers in the sequence (including the wrap-around product $x_{2018}x_1$). Show that if S is negative, then the absolute value $|\sum x_i|$ of the sum of the terms of the sequence is at most 1008.



5. If P, Q, R are distinct points on a circle, let $f(P, Q, R)$ denote the point on the circle obtained by intersecting the circle with the line through P that is parallel to the line segment QR. (If the line through P is tangent to the circle, then that is counted as a double intersection, and $f(P, Q, R)$ is defined to be equal to P.)

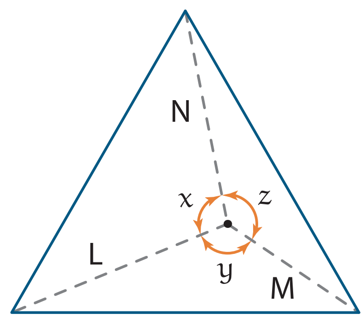
Points P_1, P_2, P_3, and P_4 lie on a circle in counterclockwise order. Define $P_5 = f(P_4, P_2, P_1)$, $P_6 = f(P_5, P_3, P_2)$, $P_7 = f(P_6, P_4, P_3)$. Prove that $P_7 = P_1$.

6. (a) Find two distinct values of $n > 3$ for which there are positive integers a, b, c such that $n = a/b + b/c + c/a$. (b) Prove that if this identity holds, then abc is a perfect cube.

The BAMO 2018 Awards Ceremony, held in March. Students were entertained by the masterful "performing applied mathematician" Tadashi Tokieda (Stanford University).

7. Let p be a point inside an equilateral triangle, joined to the vertices by line segments L, M, N. Let x, y, z be the angles made around p by those three line segments. Form a separate triangle whose sides are (congruent to) those line segments, and express its angles in terms of x, y, z.

Comment: We saw this in Stan Wagon's puzzle blog, and the problem's curious origins are described in Tanya Khovanova and Alexey Radul's *Jewish Problems*, Problem 8 (arxiv.org/abs/1110.1556).



8. Place forty 1×2 dominoes on an 8×8 chessboard without overlapping. They may partially stick out from the chessboard, but the center of each domino must be strictly inside the chessboard (not on its border). Can you place any more than forty? For any placement that you find, calculate the smallest distance from a domino center to the edge of the board.

Comment: Another problem from Gregory Galperin. We think that we know the maximum, but cannot rigorously prove it.

9. *Generous Automated Teller Machine.* You have n boxes, $b_1, b_2, \ldots, b_n$, each initially containing one coin. You are then allowed to make successive moves of the following two kinds:

- For $2 \leqslant i \leqslant n$, if b_i is nonempty, you may remove one coin from it and add two coins to b_{i-1}.
- For $3 \leqslant i \leqslant n$, if b_i is nonempty, you may remove one coin from it and interchange the entire contents of b_{i-1} and b_{i-2}.

For $n = 5$, what is the largest number of coins that you can get into b_1?

Comment: This is a "well-known" problem, due to Hans Zantema. It was on the 2010 IMO (problem 5) and the subject of several subsequent articles and blogs, including recently on Stan Wagon's blog. Our intrepid problem solver, Richard Stong, has come up with a very elegant solution for general n, using Ackermann-like functions due to Donald Knuth, that we will report on in due course.

2017 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power, and importance of mathematical ideas and ways of understanding the world.

This report acknowledges grants and gifts received from January 1–December 31, 2017. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, please let us know so we can correct our records. If your gift was received after December 31, 2017, your name will appear in the 2018 Annual Report. For more information on our giving program, please visit www.msri.org.

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