1. Let $Y_1, \ldots, Y_n$ be a random sample from the uniform density on $[0, \theta]$, where θ is an unknown parameter.

   (a) Calculate the Likelihood function $L(\theta|Y_1, \ldots, Y_n) = p(Y_1, \ldots, Y_n|\theta)$ for θ.

   (b) Let $\Omega = \{\theta : 0 < \theta \leq \theta_0\}$. Show that $\max\{L(\theta|Y_1, \ldots, Y_n) : \theta \in \Omega\} = (Y_{\max})^{-n}$, where $Y_{\max} = \max\{Y_1, \ldots, Y_n\}$.

   (c) Write out the form of the Generalized Likelihood Ratio λ for the test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta < \theta_0$.

   (d) Show that the Generalized Likelihood Ratio Test calls for H_0 to be rejected at level α if $Y_{\max} \leq \theta_0 \sqrt[n]{\alpha}$.

2. Let $(I_1, Y_1), \ldots, (I_n, Y_n)$ be i.i.d. from distribution P_θ, where $\theta = (\lambda, \mu) \in (0, 1) \times \mathbb{R}$,

$$P_\theta(I_i = 1) = \lambda = 1 - P_\theta(I_i = 0),$$

   and, given $I_i = j$, $Y_i \sim N(\mu, \sigma_j^2)$, where $\sigma_0 \neq \sigma_1$ are known positive values.

   (a) Write down the complete likelihood function $L_c(\lambda, \mu)$ assuming that all of $(I_1, Y_1), \ldots, (I_n, Y_n)$ are observed.

   (b) Give explicitly the maximum likelihood estimates of λ and μ.

   (c) Now suppose that the I_i are not observed. Give as explicitly as possible the E- and M-steps of the EM algorithm, including recursive formulae for the EM iterates $\lambda^{(k)}$ and $\mu^{(k)}$.