1. Let $Y_1, \ldots, Y_n$ be a random sample from the density $p(y|\theta) = 2y/\theta^2$, $0 < y < \theta$, where $\theta > 0$ is an unknown parameter.

   (a) Let $\overline{Y} = (1/n)\sum_{i=1}^{n} Y_i$. Find the mean and variance of $\overline{Y}$.

   (b) Show that $\widehat{\theta} = (3/2)\overline{Y}$ is an unbiased estimator of θ.

   (c) State the Cramer-Rao Inequality for the above situation.

   (d) Show that $\mathrm{Var}(\widehat{\theta})$ violates the Cramer-Rao inequality. Explain why.

2. (a) Let X have normal distribution with mean θ and variance σ^2 and let g be a differentiable function satisfying $E|(g'(X))| < \infty$. Show that

$$E[g(X)(X - \theta)] = \sigma^2 Eg'(X).$$

   (Hint: use integration by parts or Fubini's theorem.)

   (b) Let $g(x)$ be a function with $-\infty < Eg(X) < \infty$ and $g(-1)$ is finite. If X has a Poisson distribution with mean λ, show that

$$E(\lambda g(X)) = E(X(g(X - 1))).$$