

Fall 2009 Math 541a Exam

1. Let X_1, \dots, X_n be a random sample from a Bernoulli distribution with parameter $p \in (0, 1)$, that is,

$$P(X_i = 1) = p, \quad \text{and} \quad P(X_i = 0) = 1 - p.$$

- (a) Determine the UMVU of

$$q(p) = p(1 - p).$$

- (b) Prove that the odds ratio

$$q(p) = \frac{p}{1 - p}$$

is not unbiasedly estimable.

- (c) Determine necessary and sufficient conditions on $q(p)$ such that the UMVU of $q(p)$ exists.

2. Let Y_1, \dots, Y_n be independent with distribution $Y_i \sim \mathcal{N}(\theta_0 + \theta_1 x_i, 1)$, where x_1, \dots, x_n are known real numbers.

- (a) Determine the maximum likelihood estimator $(\hat{\theta}_0, \hat{\theta}_1)$ of (θ_0, θ_1) .
- (b) Calculate the Fisher information matrix for (θ_0, θ_1) .
- (c) Compare the Cramer Rao lower bound for the estimation of θ_1 when θ_0 is unknown to the case where θ_0 is known, and show this second lower bound is the smaller.
- (d) With both parameters unknown, find a simple necessary condition on a sequence of real numbers x_1, x_2, \dots such that $(\hat{\theta}_0, \hat{\theta}_1)$ is consistent for (θ_0, θ_1) .