MATH 507a **Screening Exam** **Spring, 2010**

Last Name: ____________________ **First Name:**____________________

1. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. Poisson random variables with parameter $\lambda > 0$, and let $\eta_n = \Pi_{k=1}^{n} X_k$.

(i) Show that $\{\eta_n\}_{n=1}^{\infty}$ converges to zero in probability.

(ii) Is it possible to find a subsequence $\{\eta_{n_k}\}_{k=1}^{\infty}$ and a non-zero random variable η with finite moment such that $\lim_{k\to\infty} \mathbf{E}|\eta_{n_k} - \eta| = 0$?

2. Assume that $X_1, X_2, \ldots$ are independent random variables. Show that $\sup_{n\geq 1} X_n < \infty$ a.s. if and only if

$$\sum_{n=1}^{\infty} \mathbf{P}(X_n > A) < \infty \text{ for some constant } A.$$

3. Let $X_1 X_2, \ldots$ be i.i.d. with $\mathbf{E}X_i = 0$ and $\mathrm{Var}(X_i) = \sigma^2 > 0$, and let $S_n = X_1 + \ldots + X_n$. Let N_n be a sequence of integer valued random variables independent of $X_i, i \geq 1$, and let a_n be a sequence of positive integers with $N_n/a_n \to 1$ in probability and $a_n \to \infty$ as $n \to \infty$.

What is the limit distribution of $\frac{S_{N_n}}{\sigma\sqrt{a_n}}$ as $n \to \infty$?