MATH 505a **Screening Exam** **Fall, 2009**

Last Name: ____________________ **First Name:**____________________

1. A drawer contains N pairs of socks; each sock has *precisely* one mate. The $2N$ socks are randomly arranged in the drawer. I choose k socks (randomly) from among the $2N$ socks in the drawer, with $2 \le k \le 2N$. What is the *expected* number of complete pairs in my sample of k socks?

2. A clerk in a gas station is rolling a fair dice while waiting for the customers to come. Suppose that the number of times the dice is rolled between two customers has a Poisson distribution with parameter $\lambda = 5$. Let ξ be the total points (of the dice) the clerk observed right before the next customer comes in. Determine $E\xi$ and $D\xi$ (standard deviation).

3. Let a random variable X be normal $N(\mu, \sigma^2)$ and let the conditional distribution of Y given X be normal $N(a_1 + a_2X, \sigma_1^2)$.

(a) Find the joint probability density function of X and Y.

(b) Find the marginal distribution of Y and the correlation coefficient of X and Y.

4. Let ξ and η be two random variables, both taking only two values. Show that if they are uncorrelated, then they are independent.