MATH 507a QUALIFYING EXAM February 4, 2014. One hour and 50 minutes, starting at 5pm.

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1. Let $X_1, X_2, \cdots$ be uncorrelated random variables with $E[X_i] = \mu$ and $\mathrm{Var}(X_i) \le C < \infty$. If $S_n = X_1 + \cdots + X_n$, *prove* that as $n \to \infty$, $S_n/n \to \mu$ (part a) in L^2 and (part b) in probability. For part c), say whatever you can regarding almost sure convergence, for the sequence S_n/n — maybe something about a counterexample, or about convergence along subsequences — no need to give a proof.

2. A person plays an infinite sequence of games. He wins the nth game with probability $\frac{1}{\sqrt{n}}$, independently of the other games.

(i) Prove that for any A, the probability is 1 that the player will accumulate at least A dollars if he gets \$1 each time he wins two games in a row. (Equivalently, his total winnings W satisfies $1 = P(W = \infty)$.)

(ii) Does the claim in part (i) hold if the player gets \$1 each time he wins three games in a row?

3. Let X_n be a sequence of independent r.v. such that X_n is uniformly distributed on $[0, n^2]$. Find sequences a_n, b_n such that the sequence $\left(\sum_{i=1}^n X_i - a_n\right)/b_n$ converges in distribution to a nondegenerate limit. What is that limit? [There are three tasks: part a) is to name the sequences of constants, part b) is to name the nondegenerate distribution, and part c) is to prove, or at least sketch a proof, of this distributional convergence.]