MATH 505a QUALIFYING EXAM February 6, 2014. One hour and 50 minutes, starting at 5pm.

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1. A person plays a sequence of m games. He wins the nth game with probability a_n independently of the other games. Every time that he wins *two* consecutive games, he is rewarded with \$1; let R be the total reward, so that $0 \le R \le m-1$.

(i) As a function of $a_1, a_2, \ldots, a_m$, give exact expressions for $\mathbb{E}\, R$ and $\mathrm{Var\,R}$.

(ii) Now suppose that $m = 100$ and $a_n = .1$ for all n. Simplify numerically $\mathbb{E}\, R$ and $\mathrm{Var\,R}$.

2. Assume that $X_1, X_2, \cdots$ are independent and identically distributed, each with density

$$f(x) = x/2 \text{ for } 0 < x < 2.$$

For each of the following random variables, simplify the density *or* cumulative distribution function; you may choose either one, for each random variable.

a) $S = X_1 + X_2$.

b) $L = \min(X_1, \ldots, X_{100})$.

c) $R = X_1/X_2$.

d) $M =$ the 10^{th} smallest of $X_1, \ldots, X_{100}$.

3. Let $X_1, X_2, \cdots$ be uncorrelated random variables with $E[X_i] = \mu$ and $var(X_i) \le C < \infty$. If $S_n = X_1 + \cdots + X_n$, show that as $n \to \infty$, $S_n/n \to \mu$ in probability. That is, prove that for any $\varepsilon > 0$,

$$\lim_n \mathbb{P}\left(\left| \frac{S_n}{n} - \mu \right| \ge \varepsilon \right) = 0.$$

Even if you are not comfortable with limits, simply give the best upper bound that you can, of the form

$$\mathbb{P}\left(\left| \frac{S_n}{n} - \mu \right| \ge \varepsilon \right) \le \underline{\qquad}.$$