# COMPLEX ANALYSIS GRADUATE EXAM

**Fall 2013**

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning, and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Compute
$$\int_0^\infty \frac{\log^2 x}{1+x^2}\,dx.$$

2. Find the number of *distinct* zeros of $f(z) = z^6 + (10-i)z^4 + 1$ inside $(-1,1)\times(-1,1)$.

3. Suppose that $f$ is holomorphic in a neighborhood $U$ of $a \in \mathbb{C}$. Consider the following two statements:

(i) There exist two sequences $\{z_k\}_{k=1}^\infty$ and $\{w_k\}_{k=1}^\infty$ in $U\backslash\{a\}$ converging to $a$ such that $z_k \neq w_k$ and $f(z_k) = f(w_k)$ for all $k \in \mathbb{N}$.

(ii) $f'(a) = 0$.

Determine whether either of the statements implies the other one. In each case justify your answer with a proof or a counterexample.

4. Let $f$ be analytic in an open set $U \subseteq \mathbb{C}$, and let $K \subseteq U$ be compact. Show that there exists a constant $C$ depending on $U$ and $K$ such that
$$|f(z)| \le C\left(\int_U |f|^2\right)^{1/2}$$
for all $z \in K$.