

**MATH 505a QUALIFYING EXAM September , 2013. One hour and 50 minutes**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1.) In an infinite sequence of independent trials, events  $A, B$  are mutually exclusive, with  $a = \mathbb{P}(A) > 0$  and  $b = \mathbb{P}(B) > 0$ .

a.) What is the probability that  $A$  will occur before  $B$ ?

b.) In repeated independent tosses of a pair of fair dice, what is the probability that the sum 3 will occur before the sum 7?

2.) Let  $X$  and  $Y$  be independent, standard normal. Let  $W = X + Y$  and  $Z = X - Y$ .

a.) Show that  $W$  and  $Z$  are independent.

b.) Simplify  $\mathbb{E}(X + 2Y|Z)$ .

c.) Simplify  $\mathbb{E}(X|X > 0)$ .

3.)  $n$  balls are placed into  $d$  boxes at random, with all  $d^n$  possibilities equally likely. Assume  $d > 8$ . Let  $X$  be the number of empty boxes.

a) Calculate and simplify:  $\mathbb{E} X = \underline{\hspace{2cm}}$

b) Calculate and simplify:  $\text{Var} X = \underline{\hspace{2cm}}$

c) Let  $A$  be the event that boxes 1,2,3,4 are all empty,  $B$  be the event that boxes 3,4,5,6 are all empty, and  $C$  be the event that boxes 5,6,7,8 are all empty. Compute exactly,  $\mathbb{P}(A \cup B \cup C) = \underline{\hspace{2cm}}$

d) Let  $D$  be the event that no box receives more than 1 ball. Fix  $a \in (0, 1)$ . If both  $n, d \rightarrow \infty$  together, what relation must they satisfy in order to have  $\mathbb{P}(D) \rightarrow a$ ?