1. Let $X_1, \ldots, X_n$ be a random sample from a distribution with variance $\mathrm{Var}(X_1) = \sigma^2 < \infty$, and let $T_n = T_n(X_1, \ldots, X_n)$ be some statistic.

   (a) Write down an expression for the jackknife estimator V_n of $\mathrm{Var}(T_n)$ in terms of

   $$T_{n-1,i} = T_{n-1}(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n), \quad i = 1, \ldots, n.$$

   (b) Now let $T_n = \overline{X}_n = n^{-1} \sum_{i=1}^n X_i$ be the sample mean. Show that:

      i. $\mathrm{Var}(T_n) = \sigma^2/n$

      ii. $$W_n = \frac{1}{n(n-1)} \sum_{i=1}^n (X_i - \overline{X}_n)^2$$

      is an unbiased estimator of $\mathrm{Var}(T_n)$

      iii. $V_n = W_n$

2. Given

   - an index set S;
   - a distribution $\pi = (\pi_i)$ on S, and $\pi_i > 0$ for all $i \in S$;
   - a Markov chain on S with transition matrix $\mathbf{Q} = (q_{ij})$, where $q_{ij} > 0$ for all $i \neq j$ (the reference chain).

   We construct a new Markov chain whose transition matrix $\mathbf{P} = (p_{ij})$ is given by

   $$p_{ij} = q_{ij} \frac{\pi_j \, q_{ji}}{\pi_i \, q_{ij} + \pi_j \, q_{ji}}$$

   (a) Show that

      i. This new chain is reversible.

      ii. The stationary distribution of this chain is π.

   (b) Sketch an algorithm that generates random samples whose marginal distribution is π.