1. (a) Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$ be an i.i.d. sample. Find the method of moments estimate $\hat{\lambda}_{MOM}$ and the maximum likelihood estimate $\hat{\lambda}_{MLE}$ of λ.

   (b) Is $\hat{\lambda}_{MLE}$ unbiased? Is it efficient?

   (c) Give an example of a distribution where the MOM estimate and the MLE are different.

2. (a) Prove that, for any (possibly correlated) collection of random variables $X_1, \ldots, X_k$,

$$\text{Var}\left(\sum_{i=1}^{k} X_i\right) \leq k \sum_{i=1}^{k} \text{Var}(X_i). \tag{1}$$

   (b) Construct an example with $k \geq 2$ where equality holds in (1).