COMPLEX ANALYSIS GRADUATE EXAM

Fall 2012

Answer all four questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper.

1. Evaluate the integral
$$\int_0^\infty \frac{dx}{1+x^n}, n \geq 2,$$
being careful to justify your methods.

2. Find the Laurent series expansion for
$$\frac{1}{z(z+1)}$$
valid in $\{1 < |z-1| < 2\}$.

3. Suppose that f is an entire function and that there is a bounded sequence of distinct real numbers $a_1, a_2, a_3, \ldots$ such that $f(a_k)$ is real for each k. Show that $f(x)$ is real for all real x.

4. Suppose
$$f_n(z) = \sum_{k=0}^{n} \frac{1}{k! z^k}, z \neq 0$$
and let $\varepsilon > 0$. Show that for large enough n, all the zeros of f_n are in the disk $D(0, \varepsilon)$ with center 0 and radius ε.