MATH 507a **QUALIFYING EXAM** **February 1, 2012**

Answer all three questions. Partial credit will be awarded, but in the event that you can not fully solve a problem you should state clearly what it is you have done and what you have left out. Unacknowledged omissions, incorrect reasoning and guesswork will lower your score. Start each problem on a fresh sheet of paper, and write on only one side of the paper. If you find that a calculation leads to something impossible, such as a negative probability or variance, indicate that something is wrong, but show your work anyway.

1. Let $X_1, X_2, \ldots$ be independent Poisson distributed random variables, with $\lambda_i = \mathbb{E}\, X_i$. Show that
a) If $\sum_i \lambda_i < \infty$, then $\sum_i X_i$ converges, almost surely, to a finite limit.
b) If $\sum_i \lambda_i = \infty$, then $\sum_i X_i = \infty$ almost surely.

2. Let $X_1, X_2, \cdots$ be i.i.d. with uniform distribution on $[-1, 1]$. Find the limit distribution of

$$Y_n := \frac{\sum_{i=1}^n X_i}{\sqrt{\sum_{i=1}^n |X_i|^2}}.$$

3. Let $p \in (0, 2)$ and let ξ_n, $n \geq 1$, be iid random variables. Show that the following two conditions are equivalent:

(a) With probability one, the limit

$$\lim_{n\to\infty} \frac{1}{n^{1/p}} \sum_{k=1}^n \xi_k$$

exists and is finite.

(b) $E|\xi_1|^p < \infty$ AND either $E\xi = 0$ or $p \leq 1$.