TOPICS FOR THE ALGEBRA EXAM

Students should have a good background in linear algebra, including the basic canonical forms; these topics are covered in our undergraduate course Math 471.

Groups: Review of elementary group theory, isomorphism theorems, group actions, orbits, stabilizers, simplicity of A_n, Sylows theorems, direct products and direct sums, semi-direct products and extensions of a group by an abelian group, Fundamental Theorem of Abelian Groups, solvable groups.

Fields: Relative dimensions, automorphisms, splitting fields, isomorphism extension theorem, sep- arable extensions, Galois correspondence, Fundamental Theorem of Galois Theory, principal element theorem, traces and norms, radical extensions, finite fields, cyclotomic extensions, inseparable extensions, algebraic closure.

Commutative Algebra: Localization, integral extensions, unique factorization domains, Eisenstein criterion, principal ideal domains, Noetherian rings, Hilbert basis theorem, varieties, Zariski topology, Hilbert Nullstellensatz.

Modules: Irreducible modules, torsion modules, free modules, projective modules, modules over PIDs, chain conditions, tensor products, exact sequences. Noncommutative Rings: Artinian rings, Jacobson radical, Artin-Wedderburn theorem, Maschke's theorem, Skolem- Noether theorem, division rings, Wedderburns theorem on finite division rings.

References:

D. Rotman, An introduction to the theory of groups
S. Lang, Algebra
T. Hungerford, Algebra
T.Y. Lam, Lectures on modules and rings
M. Atiyah and I.G. MacDonald, Introduction to commutative algebra
D. Dummitt and R. Foote, Abstract algebra