# Topics for the Graduate Exam in Real Analysis

*Measures*: Sigma-rings, sigma fields. Set functions and measures. Outer measure. Construction of measures on $R^n$. Variation of signed measures. Hahn decomposition theorem. Absolute continuity. Mutually singular measures. Product measures. Regular measures. Measurable functions. Signed and complex measures.

*Integration*: Definition and basic properties of integrable functions over an abstract measure space. The Riemann integral and its relation to the Lebesgue integral. Lebesgue's dominated convergence theorem and related results. Radon-Nikodym theorem. Fubini's theorem. Convolution. The n-dimensional Lebesgue integral. Polar coordinates.

*Convergence*: Almost everywhere convergence, uniform convergence, almost uniform convergence, convergence in measure and in mean. Egoroff's theorem. Lusin's theorem.

*Differentiation*: Lebesgue differentiation theorem. Maximal function. Vitali covering lemma. Bounded variation. Absolutely continuous functions. Fundamental theorem of calculus.

*Metric spaces*: Topological properties, convergence, compactness, completeness, continuity of functions.

**References**:

G.B. Folland, Real Analysis: Modern techniques and their applications
P. Halmos, Measure Theory
W. Rudin, Real and Complex Analysis