Topics for the Graduate Exam in Complex Analysis

Elementary properties of holomorphic functions: Power series representation, integral representation (Cauchy's theorem for ”nice” domains). Cauchy-Riemann equations. Taylor series, Cauchy integral formula, classification of isolated singularities, meromorphic functions. Liouville's theorem. The elementary holomorphic functions (rational functions, the exponential and logarithm functions, trigonometric functions, powers and roots).

The residue theorem and its applications: Evaluating integrals by the methods of residues, counting zeros and poles. Rouche's theorem, open mapping theorem, inverse and implicit function theorems. Methods for computing residues. Harmonic functions: Mean value property and maximum principle for harmonic and analytic functions. Realization of a real harmonic function as the real part of an analytic function (construction of the conjugate harmonic function in a simply connected domain). Poisson integral formula. Schwarz's lemma.

Limits of analytic functions: Properties carried over by uniform convergence of compact subsets, various hypotheses under which one may deduce uniform convergence on compact subsets, normal families. Conformal mapping: Local mapping properties of analytic functions, the elementary mappings (Mobius transformations, exp(z), log(z), etc.), Riemann mapping theorem.

Analytic continuation: Reflection across analytic boundaries (Schwarz reflection principle), conformal mapping of polygons to the disk, Picard's theorem.

References:

L.V. Ahlfors, Complex Analysis
J.B. Conway, Functions of One Complex Variable
W. Rudin, Real and Complex Analysis