

Topics for the Graduate Exam in Probability

Foundations of probability: Axioms of probability, distribution function, generating σ -fields, Kolmogorov's extension theorem. Principle of inclusion/exclusion. Conditional probability and independence. Random variables, distributions, joint distributions (continuous and discrete), functions of random variables and vectors.

Properties of Random Variables: Probability generating functions. Expectation, moments. Moment generating functions. Characteristic functions, inversion and continuity theorems. Basic inequalities (Cauchy-Schwarz, Chebyshev, Hölder, Jensen, Markov, power mean/Lyapunov).

Computations: Conditional distribution and density, conditional expectation given a σ -field, conditional variance, the law of total variance. Main families of discrete and continuous distributions (binomial, Cauchy, exponential, gamma, geometric, normal, Poisson, uniform) and relations among them. Multivariate normal distribution. Sums of independent random variables, convolutions.

Limit Theorems: Modes of convergence (a.s., in probability, in L_p , and in distribution) and relations among them. Theorems of Slutsky and Mann-Wald. Delta method. Convergence of expected values and moments. Borel-Cantelli lemmas. Weak and strong laws of large numbers, convergence of random series, Kolmogorov's inequality. Weak convergence, tightness; Helly-Bray and Portmanteau theorems; multidimensional weak convergence and characteristic functions. The classical Central Limit Theorem, Lindeberg's condition. Poisson approximation.

References

- P. Billingsley, Probability and Measure
- L. Breiman, Probability
- K. L. Chung, A Course in Probability Theory
- R. Durrett, Probability: Theorem and Examples
- A. Klenke, Probability Theory, especially Chapters 1–8.
- A. N. Shiriyayev, Probability, especially Chapters II–IV