

Topics for the Graduate Exam in Applied Probability

Foundations of probability: Equally likely outcomes, principles of counting, permutations, combinations. Principle of inclusion/exclusion. Conditional probability. Independence. Random variables, distributions, joint distributions (continuous and discrete), functions of random variables and vectors.

Properties of Random Variables: Expectation, moments, generating functions (for distributions of integer-valued random variables and general sequences), moment generating functions, characteristic functions (excluding continuity theorem and Bochner's theorem). Basic inequalities (Cauchy-Schwarz, Chebyshev, Hölder, Jensen, Markov, power mean/Lyapunov).

Computations: Indicators. Covariance, correlation, covariance matrix. Conditional distribution and density, conditional expectation, conditional variance, the law of total variance. Sums of independent random variables, convolutions. Main families of discrete and continuous distributions (beta, binomial, Cauchy, exponential, gamma, geometric, negative binomial, normal, Poisson, uniform) and relations among them. Multivariate normal distribution.

Limit Theorems: Convergence in probability, in L_p , and in distribution. Law of large numbers, Central Limit Theorem. Poisson approximation.

Special models: Simple random walk, reflection principle, gambler's ruin.

References

- G. R. Grimmett and D. R. Strizaker, Probability and Random Processes
- A. Klenke, Probability Theory, especially Chapters 2–8
- S. Ross, A First Course in Probability, especially Chapters 1–8
- A. N. Shirayayev, Probability, especially Chapters I and IV