

Topics for the Graduate Exam in PDE

First order equations: Method of characteristics for fully nonlinear, quasilinear, and linear cases. The Cauchy problem.

Laplace equation: Harmonic and Subharmonic functions. Mean value property. Harnack principle. Maximum principle. Liouville's theorem. Poisson formula. Green's function.

Heat equation: Cauchy problem. Energy equality. Maximum principle. Nonhomogeneous heat equation. Backward uniqueness.

Wave equation: D'Alembert's formula. Spherical means. Energy equality. Duhamel's principle. Domain of dependence.

Sobolev spaces: Weak derivatives. Embedding theorems (Gagliardo-Nirenberg and Morrey). Rellich compactness theorem. Trace theorem. H^{-1} space. Rademacher's theorem.

References:

L.C. Evans, Partial Differential Equations

G.B. Folland, Introduction to Partial Differential Equations

F. John, Partial Differential Equations